Interactive Relationships

EC 320: Introduction to Econometrics

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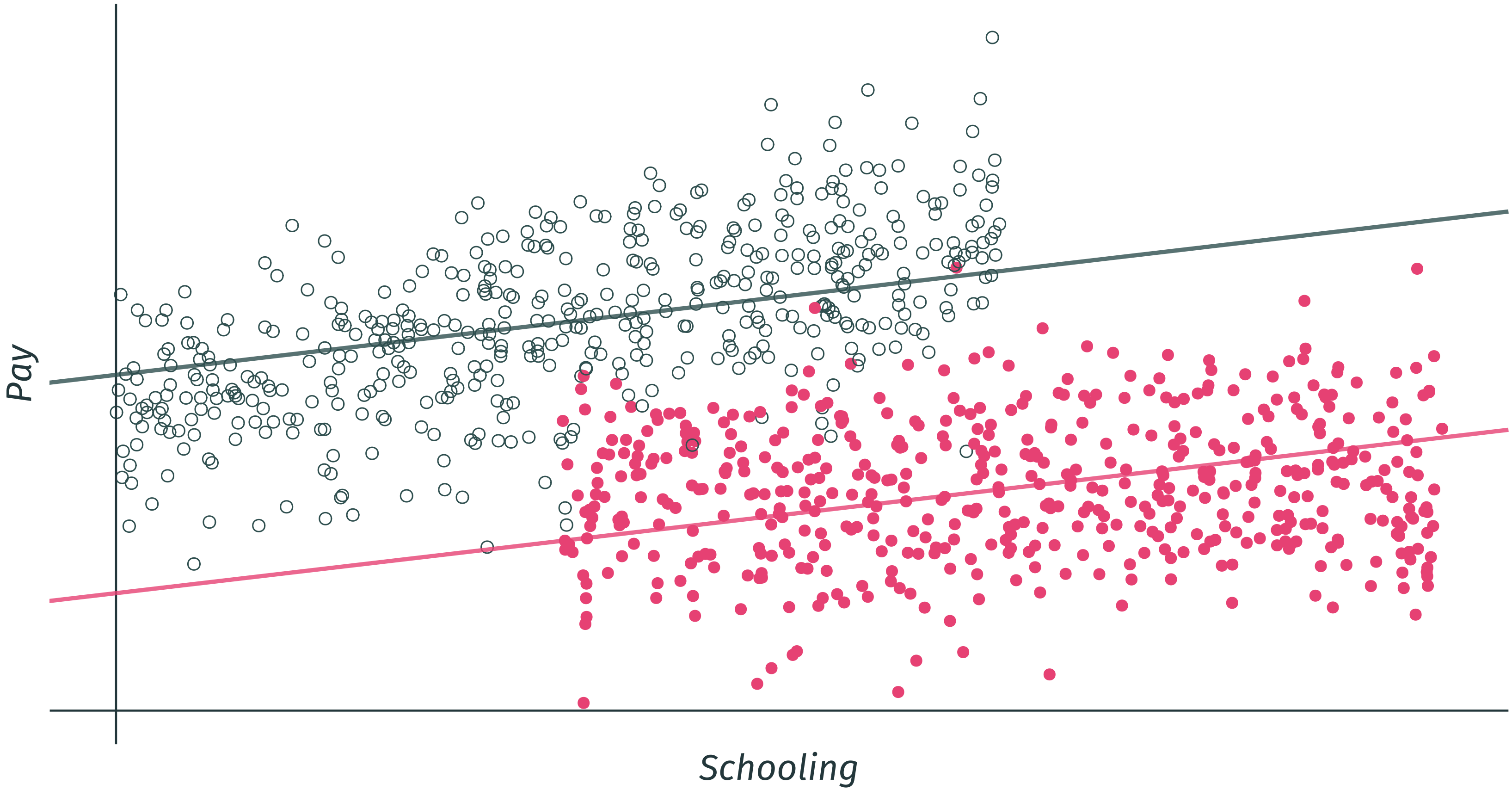
Winter 2022

Prologue

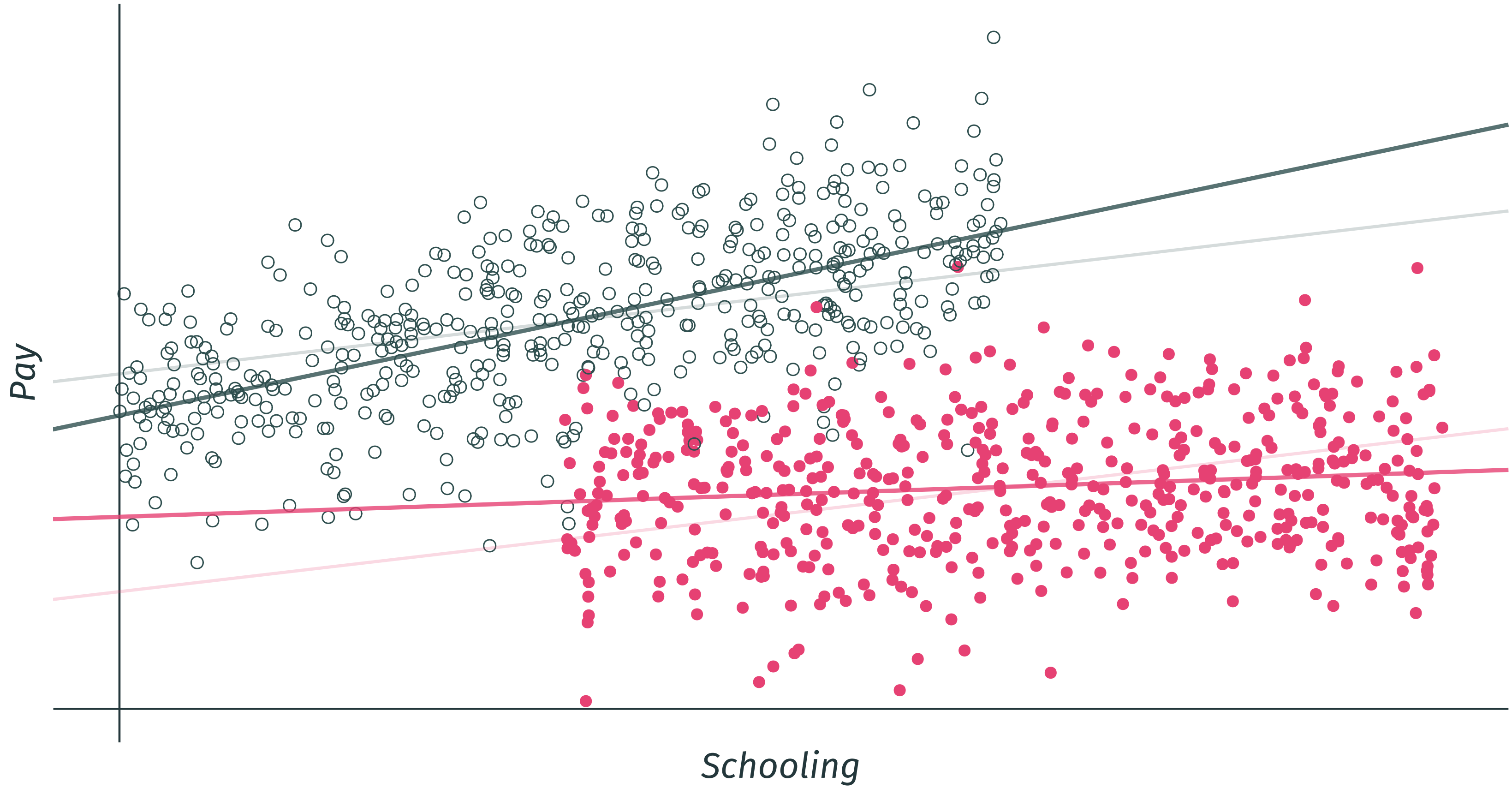
Last Time

We considered a model where schooling has the same effect for everyone (**F** and **M**):



Today

We will consider models that allow effects to differ by another variable (*e.g.*, by gender: **F** and **M**):



Interactive Relationships

Motivation

On average? For whom?

Regression coefficients describe average effects.

- Averages can mask heterogeneous effects that differ by group or by the level of another variable.

We can use **interaction terms** to model heterogeneous effects.

- Accommodate complexity and nuance by going beyond *"the effect of X on Y is β_1."*

Interaction Terms

Starting point: $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$

- X_{1i} is the variable of interest
- X_{2i} is a control variable

A richer model: Add an interaction term to study whether X_{2i} *moderates* the effect of X_{1i}:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} \cdot X_{2i} + u_i$$

Interpretation: The partial derivative of Y_i with respect to X_{1i} is the **marginal effect** of X_1 on Y_i:

$$\frac{\partial Y}{\partial X_1} = \beta_1 + \beta_3 X_{2i}$$

- Effect of X_1 depends on the level of X_2 🤯

Differential Returns to Education

Differential Returns to Education

Research Question: Do the returns to education vary by race?

Consider the interactive regression model

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + \beta_2 \text{Black}_i + \beta_3 \text{Education}_i \times \text{Black}_i + u_i$$

What is the marginal effect of an additional year of education?

$$\frac{\partial \text{Wage}}{\partial \text{Education}} = \beta_1 + \beta_3 \text{Black}_i$$

Differential Returns to Education

```
lm(wage ~ educ + black + educ:black, data = wage2) %>% tidy()
```

```
#> # A tibble: 4 x 5
#>   term        estimate std.error statistic  p.value
#>   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
#> 1 (Intercept)    196.       82.2      2.38 1.75e- 2
#> 2 educ            58.4       5.96     9.80 1.19e-21
#> 3 black          321.      263.       1.22 2.23e- 1
#> 4 educ:black     -40.7      20.7     -1.96 4.99e- 2
```

What is the **return to education** for **black** workers?

$$\left(\frac{\partial\widehat{\text{Wage}}}{\partial\text{Education}}\right)\Bigg|_{\text{Black}=1} = \hat{\beta}_1 + \hat{\beta}_3 = \mathbf{17.65}$$

Differential Returns to Education

```
lm(wage ~ educ + black + educ:black, data = wage2) %>% tidy()
```

```
#> # A tibble: 4 x 5
#>   term        estimate std.error statistic  p.value
#>   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
#> 1 (Intercept)    196.       82.2        2.38 1.75e- 2
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#> 3 black          321.      263.         1.22 2.23e- 1
#> 4 educ:black     -40.7      20.7       -1.96 4.99e- 2
```

What is the **return to education** for **non-black** workers?

$$\left(\frac{\partial \widehat{\text{Wage}}}{\partial \text{Education}}\right)\Bigg|_{\text{Black}=0} = \hat{\beta}_1 = 58.38$$

Differential Returns to Education

Q: Does the return to education differ by race?

- For answer, conduct a two-sided *t* test of the null hypothesis that the interaction coefficient equals 0 at the 5% level.

```
lm(wage ~ educ + black + educ:black, data = wage2) %>% tidy()
```

```
#> # A tibble: 4 x 5
#>   term        estimate std.error statistic  p.value
#>   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
#> 1 (Intercept)    196.       82.2      2.38 1.75e- 2
#> 2 educ            58.4       5.96     9.80 1.19e-21
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#> 4 educ:black     -40.7      20.7     -1.96 4.99e- 2
```

p-value = 0.0499 < 0.05 $\Longrightarrow$ reject null hypothesis.

A: The return to education is significantly lower for black workers.

Differential Returns to Education

We can also test hypotheses about specific marginal effects.

- *e.g.*, H_0: $\left(\frac{\partial \text{Wage}}{\partial \text{Education}} \right) \Bigg|_{\text{Black}=1} = 0.$
- Conduct a t test or construct confidence intervals.

Problem 1: `lm()` output does not include standard errors for the marginal effects.

Problem 2: The formula for marginal effect standard errors includes covariances between coefficient estimates. The math is messy.†

Solution: Construct confidence intervals using the `margins` package.

† Stay tuned.

Differential Returns to Education

The `margins` function provides standard errors and 95% confidence intervals for each marginal effect.

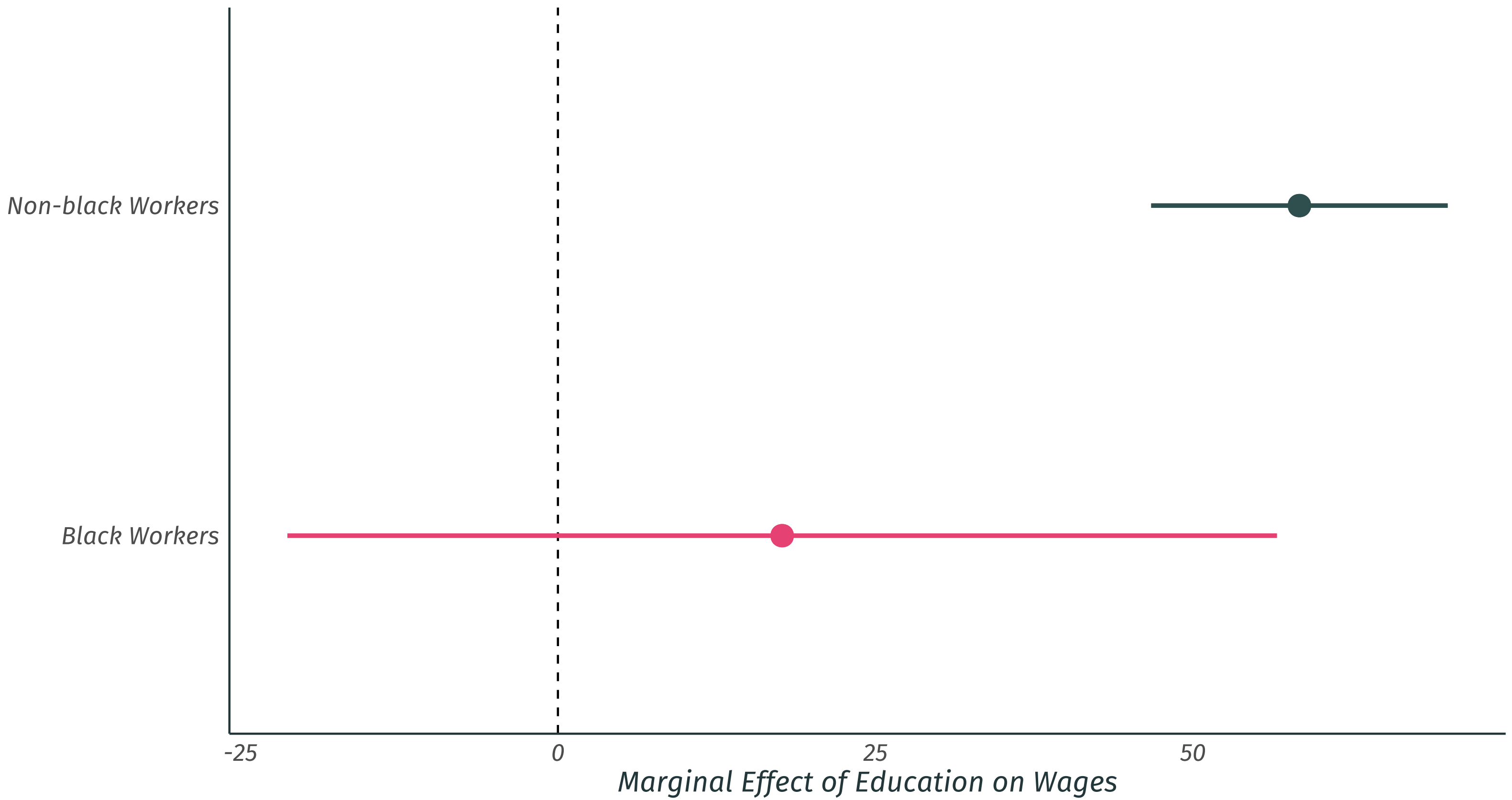
```
p_load(margins)

reg <- lm(wage ~ educ + black + educ:black, data = wage2)

margins(reg, at = list(black = 0:1)) %>% summary() %>% filter(factor == "educ")
```

```
#>   factor  black     AME      SE      z      p    lower   upper
#>     educ 0.0000 58.3773  5.9541 9.8045 0.0000  46.7074 70.0472
#>     educ 1.0000 17.6544 19.8723 0.8884 0.3743 -21.2946 56.6035
```

Marginal effect of education on wages for black workers.

Differential Returns to Education

We can use the `geom_pointrange()` option in `ggplot2` to plot the marginal effects with 95% confidence intervals.



Differential Returns to Education

We can use the `geom_pointrange()` option in `ggplot2` to plot the marginal effects with 95% confidence intervals.

```
margs <- margins(reg, at = list(black = 0:1)) %>%
  summary() %>%
  filter(factor == "educ") %>%
  mutate(Term = case_when(black == 1 ~ "Black Workers",
                          black == 0 ~ "Non-black Workers"))
margs %>%
  ggplot(aes(x = Term, y = AME, ymin = lower, ymax = upper)) +
  geom_hline(yintercept = 0, linetype = "dashed") +
  geom_pointrange() +
  coord_flip() +
  xlab("") +
  ylab("Marginal Effect of Education on Wages")
```

Differential Effects of School Funding?

Differential Effects of School Funding?

Research Question: Does the effect of school spending on student achievement vary by the share of students experiencing poverty?

- Does the marginal dollar go further in a school with a relatively affluent student body?

Regression Model

$$\textbf{Read}_i = \beta_0 + \beta_1 \textbf{Spend}_i + \beta_2 \textbf{Lunch}_i + \beta_3 \textbf{Spend}_i \times \textbf{Lunch}_i + u_i$$

- $\textbf{Read}_i$ is the average fourth grade standardized reading test score in school i (100-point scale).
- $\textbf{Spend}_i$ measured as thousands of dollars per student.
- $\textbf{Lunch}_i$ is the percentage of students on free or reduced-price lunch.

Differential Effects of School Funding?

Regression Model

$$\text{Read}_i = \beta_0 + \beta_1 \text{Spend}_i + \beta_2 \text{Lunch}_i + \beta_3 \text{Spend}_i \times \text{Lunch}_i + u_i$$

Results

```r
lm(read4 ~ spend + lunch + spend:lunch, data = meap01) %>% tidy()
```

```
#> # A tibble: 4 x 5
#>   term        estimate std.error statistic  p.value
#>   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
#> 1 (Intercept)  61.1       3.14       19.4  1.39e-76
#> 2 spend         3.29      0.601       5.47 5.13e- 8
#> 3 lunch        -0.304     0.0667     -4.56 5.53e- 6
#> 4 spend:lunch  -0.0293    0.0120     -2.44 1.49e- 2
```

Differential Effects of School Funding?

Results

```
lm(read4 ~ spend + lunch + spend:lunch, data = meap01) %>% tidy()
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```

What is the estimated marginal effect of an additional 1000 dollars per student?

$$\frac{\widehat{\partial \text{Read}}}{\partial \text{Spend}} = \hat{\beta}_1 + \hat{\beta}_3 \text{Lunch}_i$$

Differential Effects of School Funding?

Q: Does the effect of school spending on student achievement vary by the share of students experiencing poverty?

If the marginal effects do not vary by poverty levels, then

$$\begin{aligned}\frac{\partial \text{Read}}{\partial \text{Spend}} &= \beta_1 + \beta_3 \text{Lunch}_i \\ &= \beta_1\end{aligned}$$

H_0: $\beta_3 = 0$ *vs.* H_a: $\beta_3 \neq 0$

- Can evaluate using a t test or an F test.

Differential Effects of School Funding?

Conduct a two-sided *t* test at the 10% level

```
lm(read4 ~ spend + lunch + spend:lunch, data = meap01) %>% tidy()
```

```
#> # A tibble: 4 x 5
#>   term        estimate std.error statistic  p.value
#>   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
#> 1 (Intercept)  61.1       3.14       19.4  1.39e-76
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#> 3 lunch        -0.304     0.0667     -4.56 5.53e- 6
#> 4 spend:lunch  -0.0293    0.0120     -2.44 1.49e- 2
```

$\mathbf{H_0}$: $\beta_3 = 0$ *vs.* $\mathbf{H_a}$: $\beta_3 \neq 0$

t = -2.44 and $t_{0.95,\ 1823\text{-}4}$ = 1.65

Reject $\mathbf{H_0}$ if $|t|$ = $|$-2.44$|$ > $t_{0.95,\ 1823\text{-}4}$ = 1.65.

Statement is true ⇒ **reject $\mathbf{H_0}$** at the 10% level.

Differential Effects of School Funding?

Conduct an *F* test at the 10% level

```
reg_unrestrict <- lm(read4 ~ spend + lunch + spend:lunch, data = meap01)
reg_restrict <- lm(read4 ~ spend + lunch, data = meap01)
anova(reg_unrestrict, reg_restrict)
```

```
#> Analysis of Variance Table
#>
#> Model 1: read4 ~ spend + lunch + spend:lunch
#> Model 2: read4 ~ spend + lunch
#>   Res.Df    RSS Df Sum of Sq      F  Pr(>F)
#> 1   1819 408262
#> 2   1820 409596 -1     -1334 5.9434 0.01487 *
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\mathbf{H_0}$: $\beta_3 = 0$ *vs.* **$\mathbf{H_a}$:** $\beta_3 \neq 0$

p-value = 0.01487 < 0.1 ⟹ **reject $\mathbf{H_0}$** at the 10% level.

Differential Effects of School Funding?

Q: Is there a statistically significant effect of spending on student achievement for every level of poverty?

One way to answer this question is to construct confidence intervals for the marginal effects.

- Requires standard errors.
- Standard errors will depend on the poverty level (our proxy: $\mathbf{Lunch}_i$).

Time for math! 🎉

Step 1: Derive the estimated marginal effects.

$$\frac{\partial \widehat{\text{Read}}}{\partial \text{Spend}} = \hat{\beta}_1 + \hat{\beta}_3 \text{Lunch}_i$$

Step 2: Derive the variances of the estimated marginal effects.

$$\begin{aligned}
&\text{Var}\left(\frac{\partial \widehat{\text{Read}}}{\partial \text{Spend}}\right) \\
&\quad = \text{Var}\left(\hat{\beta}_1 + \hat{\beta}_3 \text{Lunch}_i\right) \\
&\quad = \text{Var}\left(\hat{\beta}_1\right) + \text{Var}\left(\hat{\beta}_3 \text{Lunch}_i\right) + 2 \cdot \text{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3 \text{Lunch}_i\right) \\
&\quad = \text{Var}\left(\hat{\beta}_1\right) + \text{Lunch}_i^2 \cdot \text{Var}\left(\hat{\beta}_3\right) + 2 \cdot \text{Lunch}_i \cdot \text{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right) \\
&\quad = \text{SE}\left(\hat{\beta}_1\right)^2 + \text{Lunch}_i^2 \cdot \text{SE}\left(\hat{\beta}_3\right)^2 + 2 \cdot \text{Lunch}_i \cdot \text{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right)
\end{aligned}$$

Step 3: Derive the standard errors of the estimated marginal effects.

$$
\begin{aligned}
\mathrm{SE}\left(\frac{\widehat{\partial \mathrm{Read}}}{\partial \mathrm{Spend}}\right)
&= \mathrm{Var}\left(\frac{\widehat{\partial \mathrm{Read}}}{\partial \mathrm{Spend}}\right)^{1/2} \\
&= \sqrt{\mathrm{SE}\left(\hat{\beta}_1\right)^2 + \mathbf{Lunch}_i^2 \cdot \mathrm{SE}\left(\hat{\beta}_3\right)^2 + 2 \cdot \mathbf{Lunch}_i \cdot \mathbf{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right)}
\end{aligned}
$$

Step 4: Calculate the bounds of the confidence interval.

$$
\begin{aligned}
&\hat{\beta}_1 + \hat{\beta}_3 \cdot \mathbf{Lunch}_i \\
&\qquad \pm t_{\text{crit}} \cdot \sqrt{\mathrm{SE}\left(\hat{\beta}_1\right)^2 + \mathbf{Lunch}_i^2 \cdot \mathrm{SE}\left(\hat{\beta}_3\right)^2 + 2 \cdot \mathbf{Lunch}_i \cdot \mathbf{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right)}
\end{aligned}
$$

Differential Effects of School Funding?

Confidence Interval

$$\hat{\beta}_1 + \hat{\beta}_3 \cdot \mathbf{Lunch}_i \pm t_{\text{crit}} \cdot \sqrt{\mathbf{SE}\left(\hat{\beta}_1\right)^2 + \mathbf{Lunch}_i^2 \cdot \mathbf{SE}\left(\hat{\beta}_3\right)^2 + 2 \cdot \mathbf{Lunch}_i \cdot \mathbf{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right)}$$

Notice that $\mathbf{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right)$ is not reported in a regression table

- Located in the variance-covariance matrix inside `lm()` object (beyond the scope of this class).
- Can't calculate by hand without about $\mathbf{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right)$.
- Special case: $\hat{\beta}_1$ and $\hat{\beta}_3$ are statistically independent $\Rightarrow$ $\mathbf{Cov}\left(\hat{\beta}_1,\ \hat{\beta}_3\right) = \mathbf{0}$.

Differential Effects of School Funding?

We can use the `cplot` function from `margins` with `ggplot2` to plot the marginal effects with 95% confidence intervals.

Differential Effects of School Funding?

We can use the `cplot` function from `margins` with `ggplot2` to plot the marginal effects with 95% confidence intervals.

```
# run regression
reg <- lm(read4 ~ spend + lunch + spend:lunch, data = meap01)

# retrieve marginal effects with 95% CI
margs <- cplot(reg, x = "lunch", dx = "spend",
               what = "effect", draw = FALSE)

# plot the marginal effects
margs %>%
  ggplot(aes(x = xvals)) +
  geom_line(aes(y = yvals)) +
  geom_line(aes(y = upper), linetype = 2) +
  geom_line(aes(y = lower), linetype = 2) +
  geom_hline(yintercept = 0, linetype = 3) +
  xlab("Percentage on Free or Reduced-Price Lunch") +
  ylab("Marginal Effect of Spending on Reading Scores")
```

Moving to Opportunity

Moving to Opportunity

Background

Policy Question: How can we lift people out of poverty?

Research Agenda: What kinds of social assistance programs have lasting effects on upward mobility?

Economists study a variety of state and federal social assistance programs.

- Medicaid, SNAP (food stamps), TANF (cash welfare), WIC (benefits for mothers), National School Lunch Program, public housing, Section 8 (housing vouchers), *etc.*
- Considerable variation in benefits and incentive structures.
- Today: Section 8 *v.s.* public housing.

Moving to Opportunity

Experiment

Research Question: Does moving from a public housing project to high-opportunity neighborhood improve well-being?

Social Experiment: Moving to Opportunity (MTO)

4600 low-income families living in federal housing projects.

- Recruited by the Department of Housing and Urban Development during the mid-1990s.
- Housing projects in Baltimore, Boston, Chicago, Los Angeles, and New York.
- Randomly assigned various forms of housing assistance.

Moving to Opportunity

Experiment

Experimental Design

Participants randomly assigned into one of three treatments:

- **Experimental group:** Housing voucher for low-poverty neighborhoods only + counseling
- **Section 8 group:** Housing voucher for any neighborhood + no counseling
- **Control group:** No housing voucher + no counseling (*i.e.*, regular public housing)

Moving to Opportunity

Experiment

Initial Results

1. Most families in the treatment groups actually used vouchers to move to better neighborhoods.
2. Improvements in physical and mental health.
3. No significant improvements in earnings or employment rates for parents.

Experiment

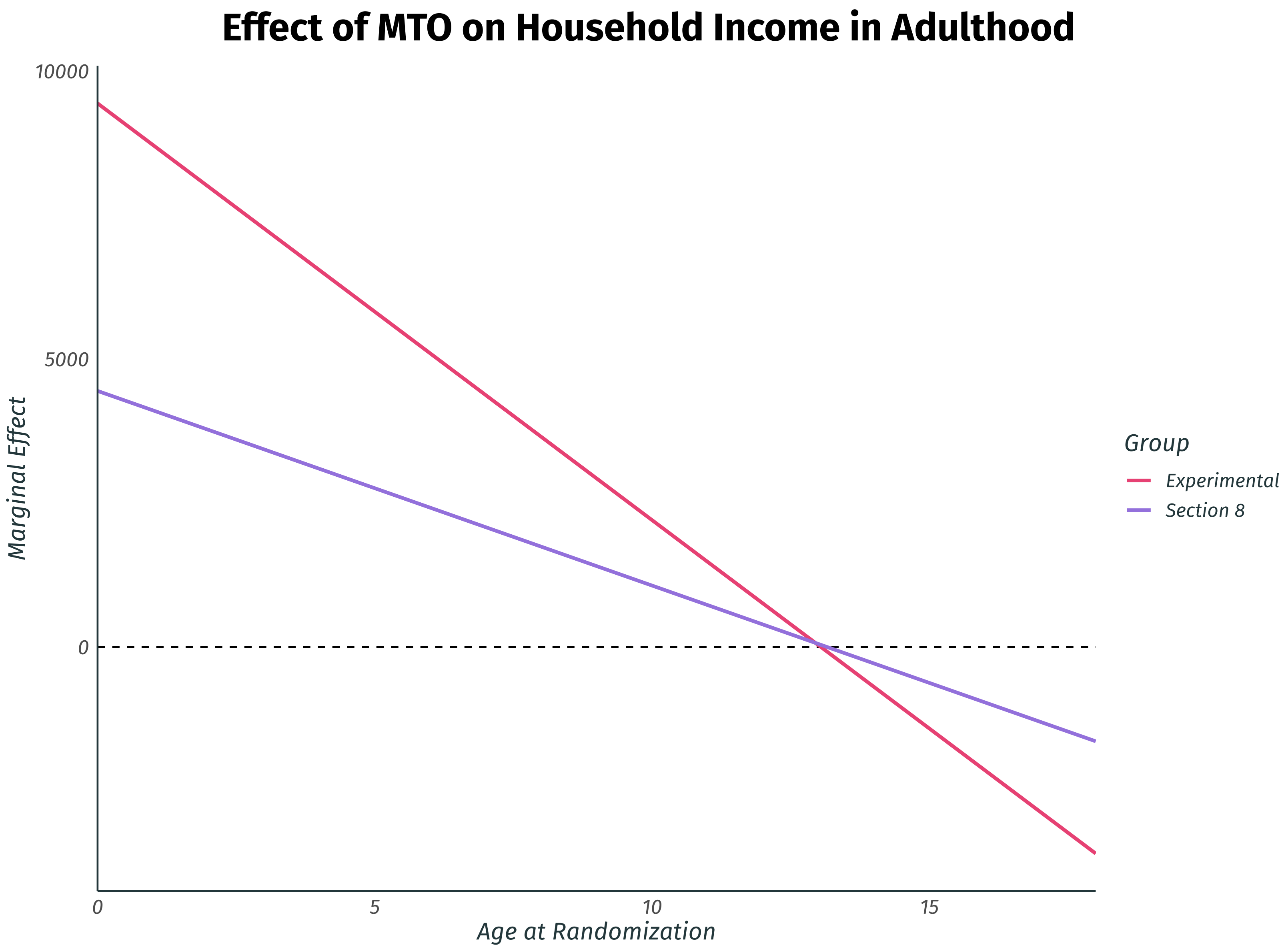
What about children?

Chetty, Hendren, and Katz (*American Economic Review*, 2016) study the long-run impact of MTO on children.

- Individual tax data linked to children from original MTO sample.
- Adulthood outcomes: income, marriage, poverty rate in neighborhood of residence, taxes paid, *etc*.
- Test how effects vary by age of child when family received voucher.

Long-Run Effects of MTO Experiment

	Household Income ($)	Married (%)	Neighborhood Poverty (%)	Taxes Paid ($)
Experimental	**9441.1**	8.309	-4.371	831.2
	(3035.8)	(3.445)	(1.770)	(279.4)
Section 8	**4447.7**	7.193	-1.237	521.7
	(3111.3)	(3.779)	(2.021)	(287.5)
Experimental × Age at Randomization	**-723.7**	-0.582	0.261	-65.81
	(255.5)	(0.290)	(0.139)	(23.88)
Section 8 × Age at Randomization	**-338**	-0.433	0.0109	-42.48
	(266.4)	(0.316)	(0.156)	(24.85)
Control Group Mean	**16259.9**	6.6	23.7	627.8
Observations	**20043**	20043	15798	20043


Effect of MTO on Household Income in Adulthood
10000
5000
0
Marginal Effect
0
5
10
15
Age at Randomization
Group
Experimental
Section 8

Long-Run Effects of MTO Experiment

	Household Income ($)	Married (%)	Neighborhood Poverty (%)	Taxes Paid ($)
Experimental	9441.1	**8.309**	-4.371	831.2
	(3035.8)	**(3.445)**	(1.770)	(279.4)
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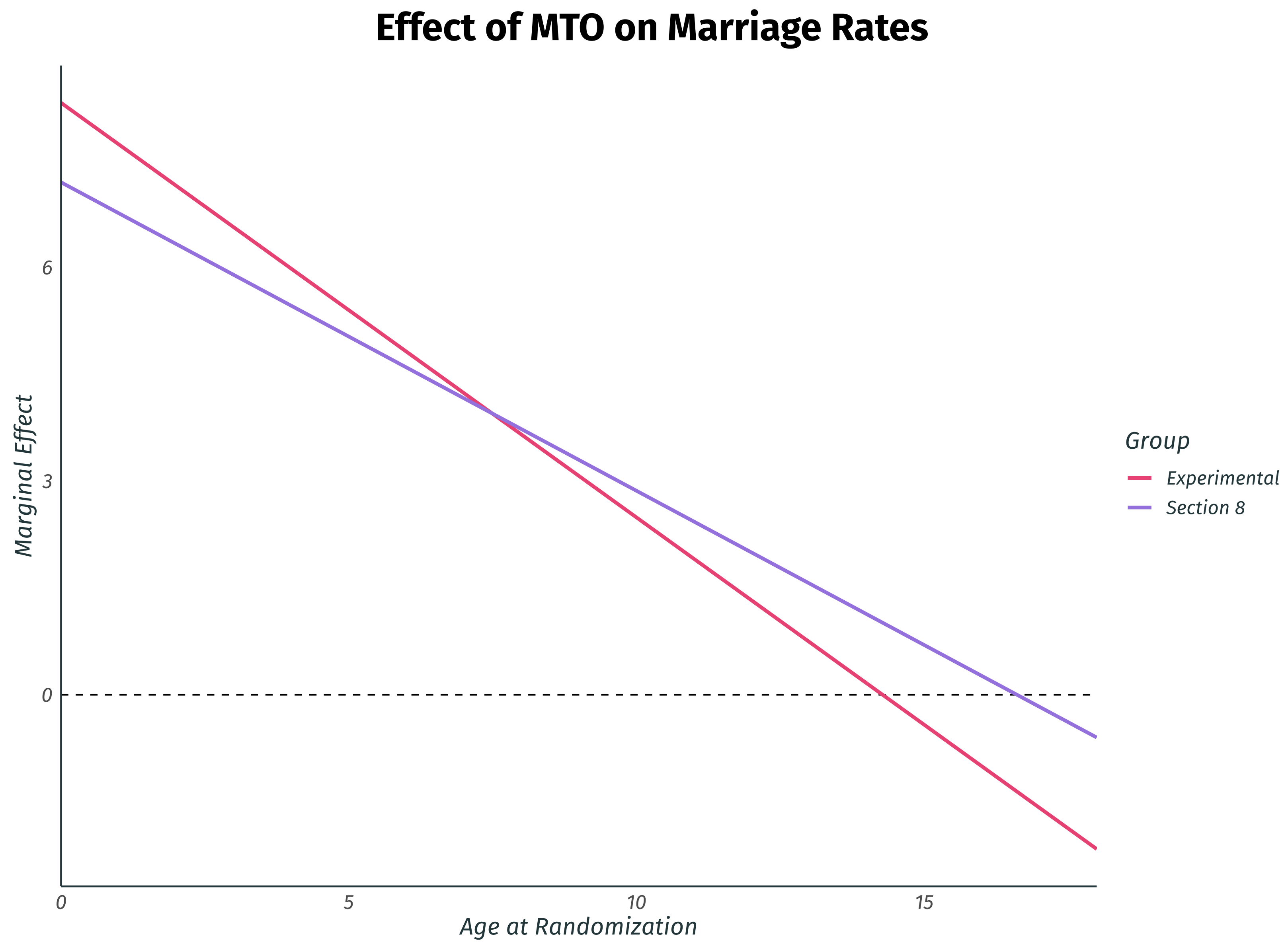
Effect of MTO on Marriage Rates
Marginal Effect
6
3
0
0
5
10
15
Age at Randomization
Group
Experimental
Section 8

Long-Run Effects of MTO Experiment

	Household Income ($)	Married (%)	Neighborhood Poverty (%)	Taxes Paid ($)
Experimental	9441.1	8.309	**-4.371**	831.2
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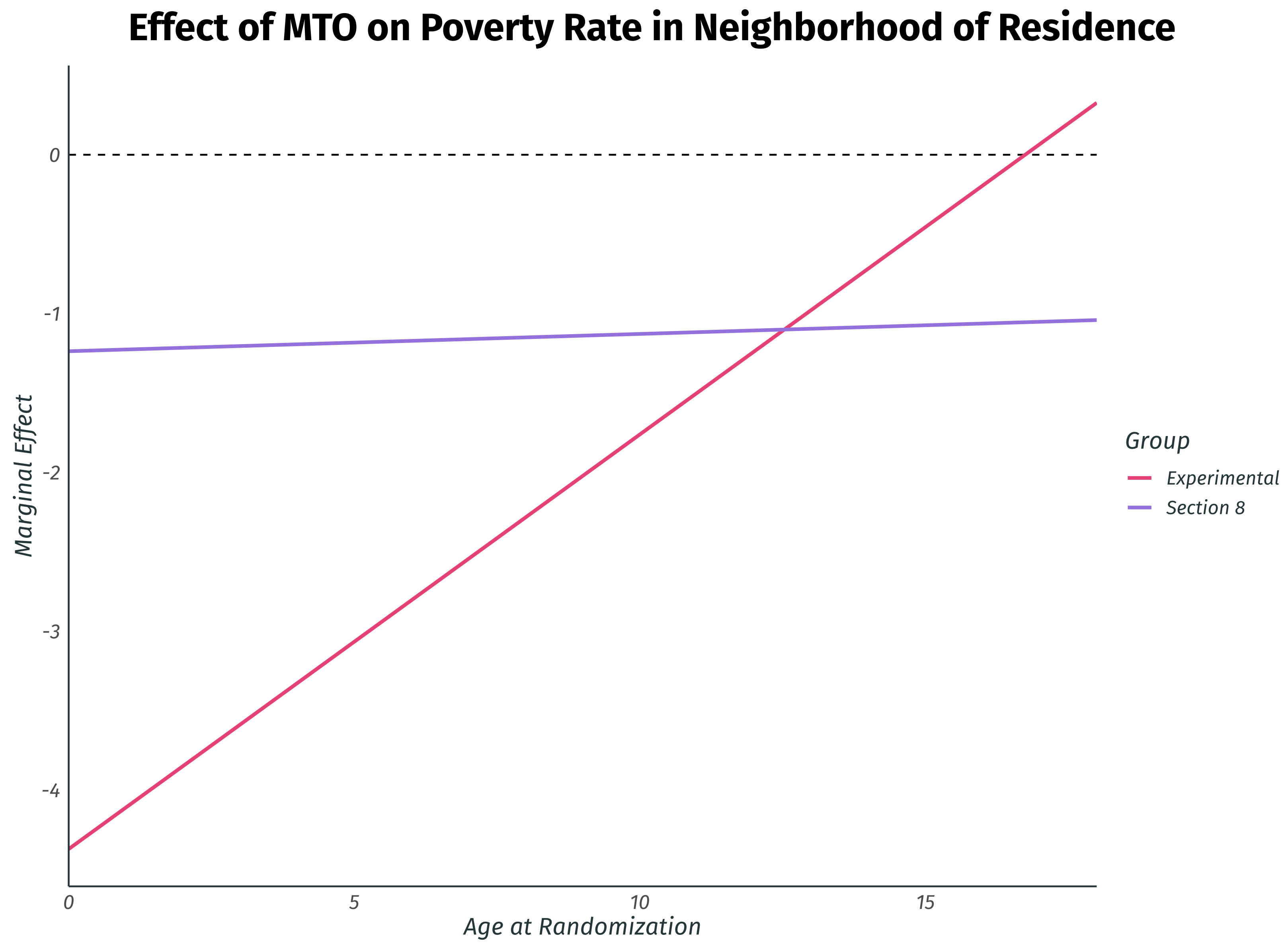
Effect of MTO on Poverty Rate in Neighborhood of Residence
0
-1
-2
-3
-4
Marginal Effect
0
5
10
15
Age at Randomization
Group
Experimental
Section 8

Long-Run Effects of MTO Experiment

	Household Income ($)	Married (%)	Neighborhood Poverty (%)	Taxes Paid ($)
Experimental	9441.1	8.309	-4.371	**831.2**
	(3035.8)	(3.445)	(1.770)	**(279.4)**
Section 8	4447.7	7.193	-1.237	**521.7**
	(3111.3)	(3.779)	(2.021)	**(287.5)**
Experimental × Age at Randomization	-723.7	-0.582	0.261	**-65.81**
	(255.5)	(0.290)	(0.139)	**(23.88)**
Section 8 × Age at Randomization	-338	-0.433	0.0109	**-42.48**
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Control Group Mean	16259.9	6.6	23.7	**627.8**
Observations	20043	20043	15798	**20043**

Effect of MTO on Taxes Paid