Lecture 9

Hedonics: Property value models

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AEM 6510

Roadmap

- What can we use to infer the demand for environmental goods?
- What do housing prices tell us?
- When do changes in house prices give us welfare measures

Environmental quantity changes

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We can't compute CS, EV, CV, etc!

Revealed preference approaches

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If there are changes in the environmental good, holding everything else fixed, that should be reflected in *some way* in changes in the price of the related private good

This change in price can tell us something about how people value the change in the environmental good

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What does this price change mean?

Hedonics: Property value models

Common market goods to use for revealed preference valuation are **properties**

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- Rooms
- Bathrooms
- School quality
- Environmental quality

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Homes located in pristine areas are likely to be more valuable than identical homes located near toxic facilities

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Property purchases are large and consequential: buyers and sellers are likely to be well-informed

It is uncontroversial that property values should reflect local attributes

The hedonic model

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Suppose that we have some quality-differentiated good (i.e. a home)

This good is characterized by a set of J property characteristics x

- parcel size, school quality, bedrooms, etc

It is also characterized by an environmental good q

The hedonic model

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Here we will assume the supply of houses is fixed in the short run so the price function arises from buyer behavior

The hedonic model: consumer's choice problem

Households buy a single property given their budget constraint and their preferences

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Here we will assume that households are effectively just choosing (x, q) instead of a specific house with the following objective:

$$\max_{x,q,z} U(x, q, z; s) \quad s.t. \quad y = z + P(x, q)$$

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$$\max_{x,q,z} U(x, q, z; s) \quad s.t. \quad y = z + P(x, q)$$

- z is the numeraire good (spending on other private goods)
- y is income
- s is the set of the household's characteristics like family size

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Another is that you just can't purchase some sets of x (i.e. a huge lot in downtown manhattan with a farm)

We won't touch on this in class but there is a **discrete choice** literature that works to alleviate these issues

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We are thus also implicitly assuming q varies across space so that households can sort into areas they prefer

- q is really picking up local environmental goods

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For homeowners we are basically assuming they rent from themselves every year

The hedonic model: consumer's choice problem

$$\max_{x,q,z} U(x,q,z;s) \quad s.t. \quad y = z + P(x,q)$$

The FOCs for this problem are:

$$\begin{aligned}
\frac{\partial U}{\partial x_j} &= \lambda \frac{\partial P}{\partial x_j} \quad j = 1, \ldots, J \\
\frac{\partial U}{\partial q} &= \lambda \frac{\partial P}{\partial q} \\
\frac{\partial U}{\partial z} &= \lambda
\end{aligned}$$

Next, combine the last two FOCs

The hedonic model: consumer's choice problem

$$\begin{aligned} \frac{\partial U}{\partial q} &= \lambda \frac{\partial P}{\partial q} \\ \frac{\partial U}{\partial z} &= \lambda \end{aligned}$$

gives us that

$$\frac{\partial P}{\partial q} = \frac{\partial U}{\partial q} \bigg/ \frac{\partial U}{\partial z}$$

At a utility-maximizing choice, a household equates their MRS between q and z and the marginal implicit cost of q

The hedonic model: consumer's choice problem

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Recall that z is the numeraire good so we can think of it in terms of dollars

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This means that $\frac{\partial U}{\partial q} \big/ \frac{\partial U}{\partial z}$ is the WTP for q, the reduction in income needed to compensate for an additional unit of q

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Knowledge of the hedonic price function P is enough to tell us about household WTP for q!

The hedonic model: bid functions

Now let's dive deeper by looking at some reference utility level $\bar{u}$:

$$U(x, q, z; s) = \bar{u}$$

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Next we will define something called a **bid function** $b(x, q, y, s, \bar{u})$ where:

$$U(x, q, y - b(x, q, y, s, \bar{u}); s) = \bar{u}$$

The bid function b is the maximum amount the household is willing to pay for:

- A house with characteristics x, q
- Given income y and household characteristics s
- Holding utility fixed

The hedonic model: bid functions

$$U(x, q, z; s) = \bar{u}$$

We can also invert this to solve for z:[1]

$$z = U^{-1}(x, q, \bar{u}, s)$$

[1] We can do this because U is monotonically increasing in z

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Income, the bid function and z are related by:

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Now we have everything we need to derive a marginal WTP function for q

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The hedonic model: deriving MWTP

$$U(x, q, y - b(x, q, y, s, \bar{u}); s) = \bar{u}$$

Differentiate with respect to q to get:

$$\frac{\partial U}{\partial q} + \frac{\partial U}{\partial z}\frac{\partial b}{\partial q} = 0$$

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We can then rearrange to get:

$$\frac{\partial b}{\partial q} = \frac{\partial U}{\partial q} \Big/ \frac{\partial U}{\partial z}$$

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Recall that the bid function is separable in income:

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Conditional on x, this defines our **compensated inverse demand function** for q!

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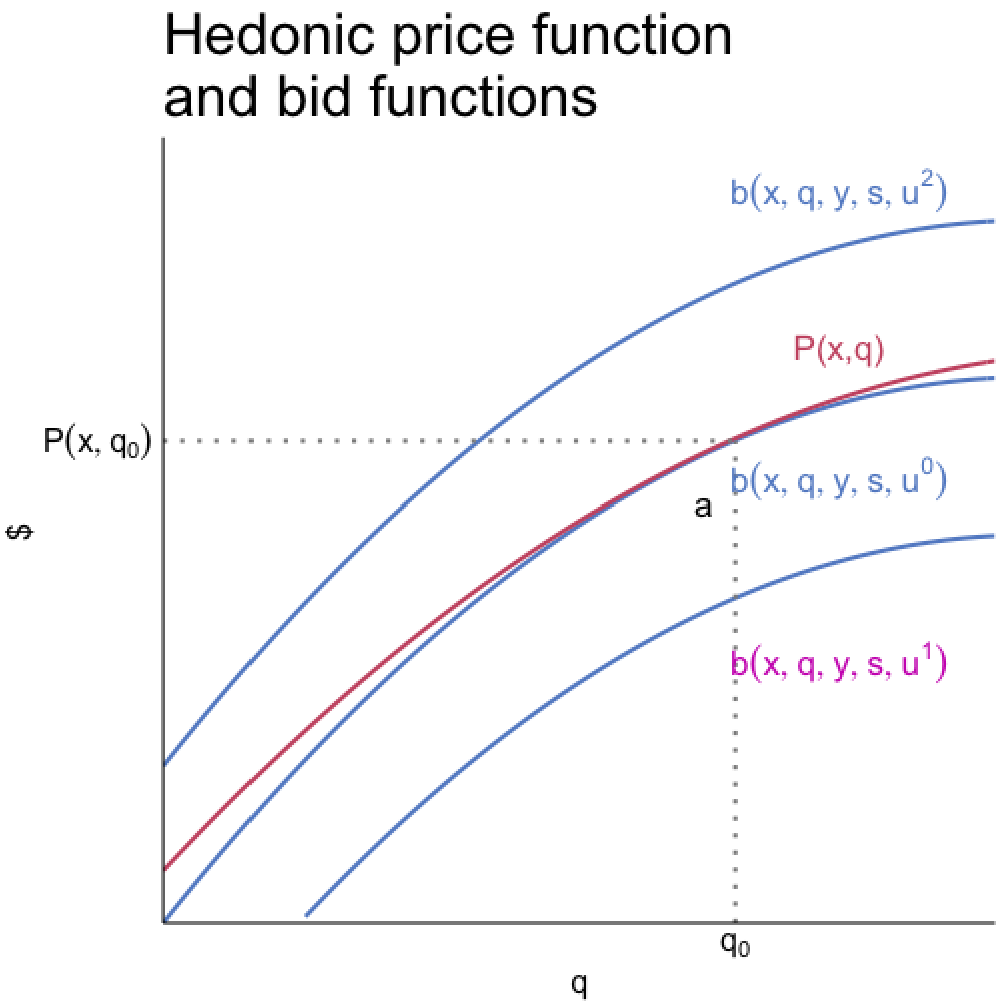
Our ultimate empirical goal is to estimate $\pi^q(x, q, s, \bar{u})$

Bid functions and housing prices

The red line is the hedonic price function

The blue lines are a single household's bid functions at different reference utility levels where $u_1 > u_0 > u_2$

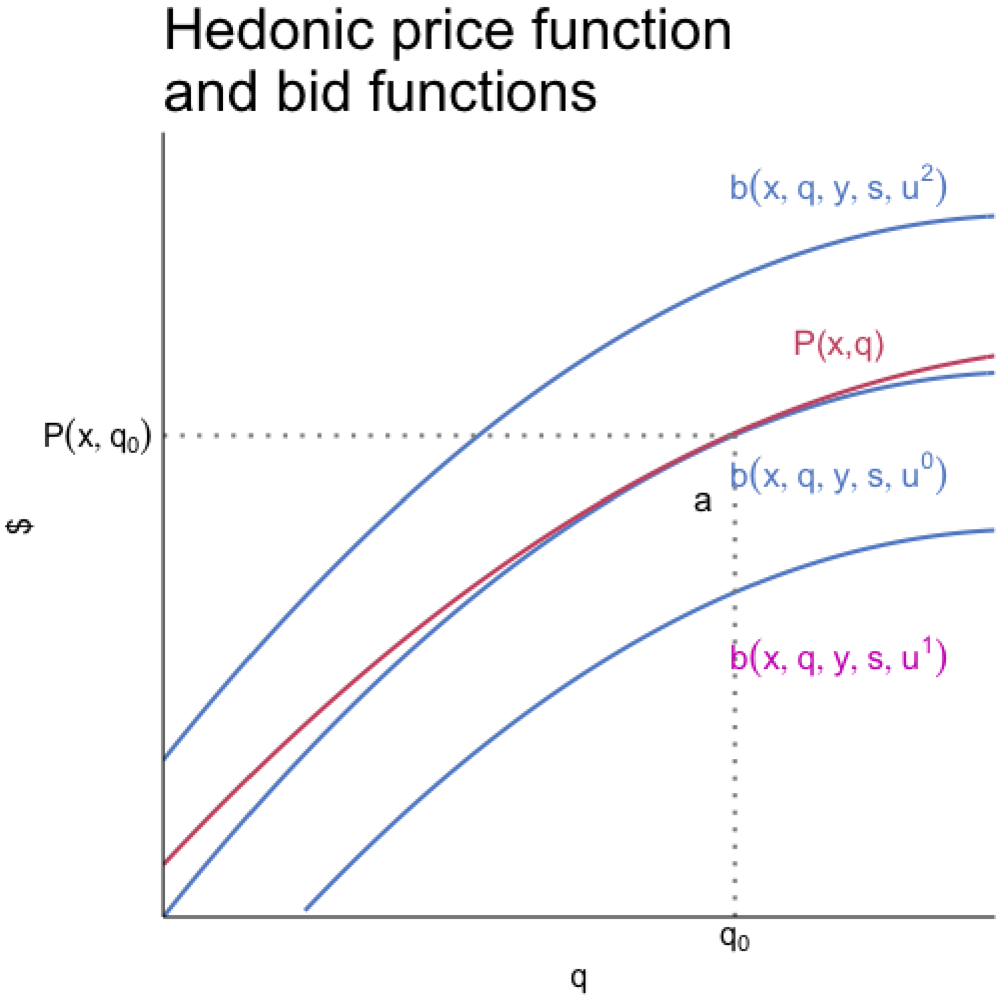
Higher utility $\rightarrow$ lower bids because same level of q can be achieved with higher z



Bid functions and housing prices

Optimal choice is where the household's bid function is tangent to the hedonic price schedule: a

This gives us an observed consumption level q_0, observed price $P(x, q_0)$, and realized utility u^0

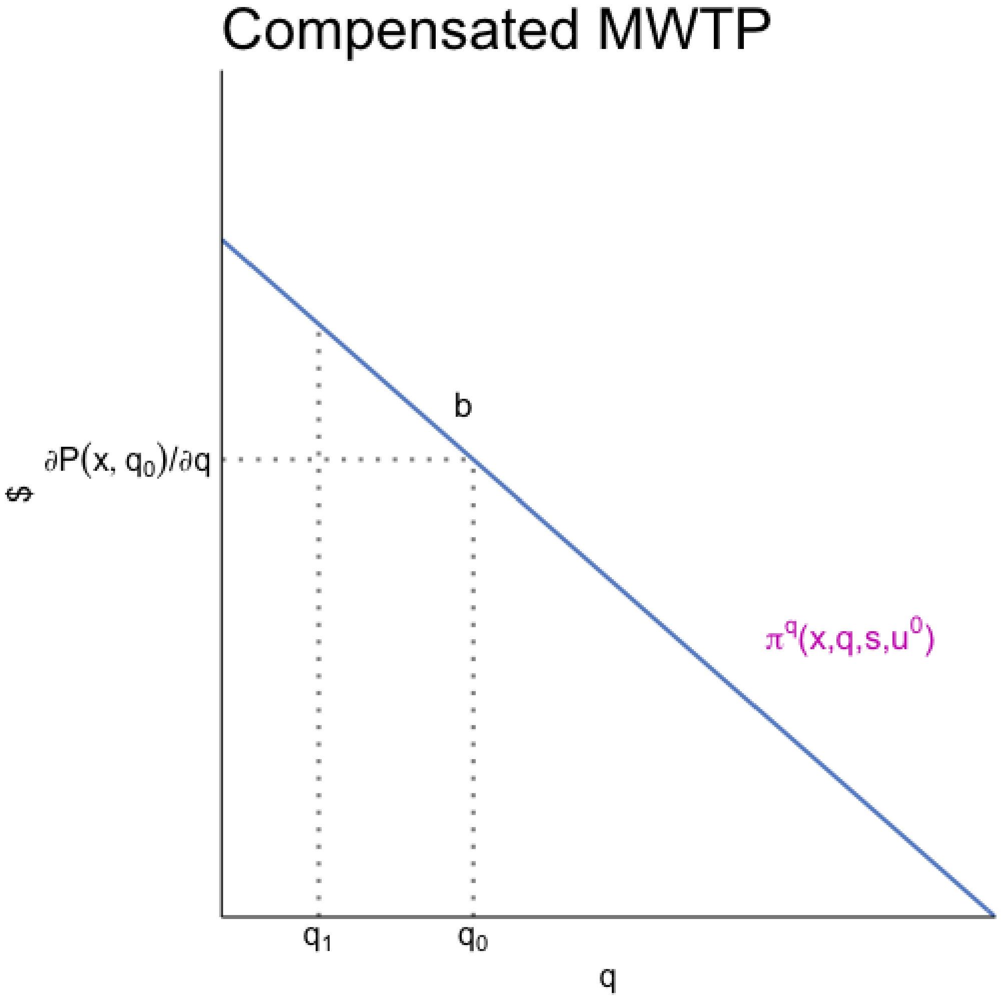


Compensated MWTP

This plot shows the corresponding compensated MWTP curve associated with $b(x, q, y, s, u^0)$

It is the slope of the bid function as q changes

We observe b if we can estimate $P(x, q)$ and its derivative



Compensated MWTP

We can estimate $P(x, q)$ using home sales prices and home attributes data

The slope of $P(x, q)$ is then equal to the MWTP for q

This gives us the consumers inverse demand for q

$$\frac{\partial P(x, q_0)}{\partial q} = \pi^q(x, q_0, s, u^0)$$