Lecture 7

Environmental policy with pre-existing distortions

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Roadmap

So far we have looked at single sector economies with:

- Pollution distortions
- Competitive markets
- Market power distortions

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- Pollution distortions
- Competitive markets
- Market power distortions

Now we will learn about multi-sector economies

How does environmental policy spillover into these other sectors?

How does environmental policy interact with revenue-raising taxes (e.g. income taxes)?

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- There is a representative (single) firm
- There is a representative household

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This allows us to treat individual and aggregate behavior the same

1: The underlying critical assumption is that utility and profit functions take what's called a Gorman form.

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- *X* is consumption of the polluting good
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- N is the hours of leisure time
- E is aggregate emissions

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where $U_{XX}, U_{NN} < 0$ and $U_{XX}U_{NN} - U_{NX}^2 > 0$ and the person is endowed with some amount of time T to allocate between work and leisure

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Household income is then: $w \cdot (T - N)$

We can now write the households utility maximization problem as:

$$\max_{X,N,Z} U(X,Z,N,E) = U(X,N) + Z - D(E)$$

subject to:
$$w \cdot (T - N) = Z + pX$$

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with FOCs:

$$U_X=p \qquad U_N=w$$

which implicitly define the demand function for consumption X(p,w) and the demand function for leisure N(p,w)

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We have two equations and two unknowns so we can solve to get:

$$rac{\partial N}{\partial p} = rac{-U_{XN}}{U_{XX}U_{NN} - U_{XN}^2} \qquad rac{\partial X}{\partial p} = rac{U_{NN}}{U_{XX}U_{NN} - U_{XN}^2}$$

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If X and N are substitutes, $-U_{XN}>0$, and leisure increases in the price of the consumption good

If they are complements, $-U_{XN} < 0$, and leisure decreases in the price of the consumption good

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If Y is going on a picnic and Y is video games: Y and Y are substitutes

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 where $E = \delta X$

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- $\delta = 1$ so we can use E and X interchangeably
- $C'(X) > 0, C''(X) \ge 0$
- The polluting industry's demand for labor is small relative to the entire economy, i.e. wages are effectively fixed for the household

Now lets solve for the social optimum:

$$\max_{X} W = \underbrace{U(X,N) + w \cdot (T-N) - pX - D(X)}_{\text{Consumer Utility}} + \underbrace{pX - C(X)}_{\text{Firm profit}}$$

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One way you can think about this is as if the regulator imposes a quantity standard X^{st} and then a market price p^{st} arises which affects leisure demand

The FOC for the optimum is:

$$[U_X-D'(X)-C'(X)+[U_N-w]rac{\partial N}{\partial X}=0]$$

where the last term captures the households indirect leisure response to the regulator's policy choice

Environmental policy with leisure

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Marginal abatement cost $(U_X - C'(X))$ equals marginal damage (D'(X))!

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The consumer's utility maximization problem is:

$$\max_{X,Z,N} U = u(X,N) + Z - D(E)$$
subject to $(1-m)w(T-N) = Z + pX$

Where the budget is scaled down by (1-m) reflecting the income tax

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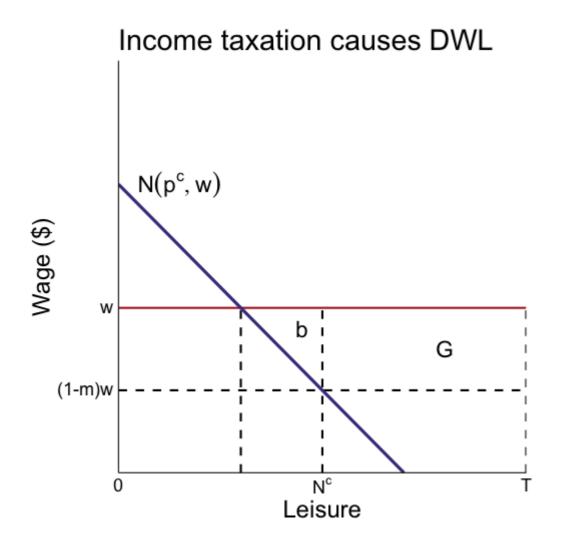
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The tax m makes the consumer act as if there is a subsidy mw on leisure

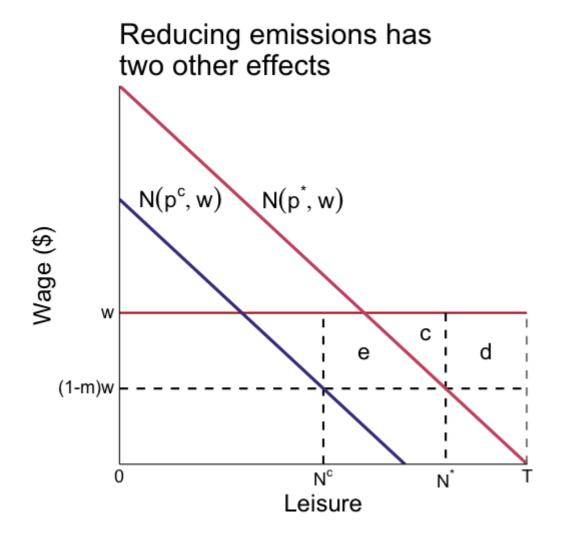


 \boldsymbol{w} is the perfectly elastic demand for labor

 N^c is how much leisure the consumer chooses, since (1-m)w < w this is too much and induces DWL equal to b

This is called excess burden

The tax raises revenues equal to G: $mw imes (T-N^c)$

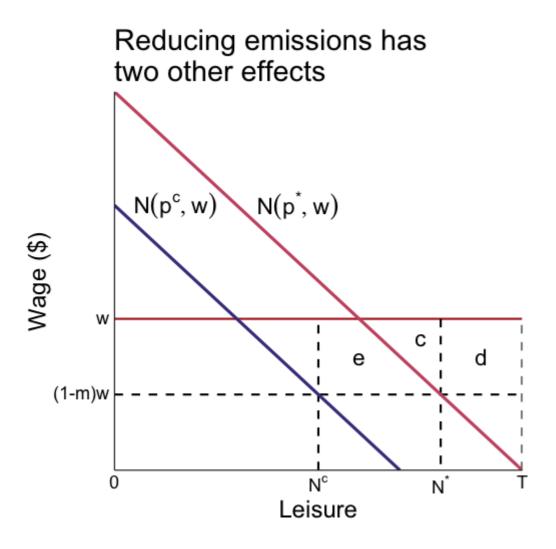


Suppose N and X are substitutes, and the regulator sets $X=X^*$ where $X^* \to MAC = MD$

This raises the price of X, shifts leisure demand to the right

New DWL is c, and government revenues are now only d

Change in DWL from $X^c \to X^*$ is indeterminant



Fixing the pollution externality had two effects:

- 1. Indeterminant effect on the distortion in the labor market
- 2. Reduced the amount of revenue the government raised through labor taxation

Second-best environmental policy

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First let's consider the case where they can only raise revenue via a labor tax: this is non-revenue raising environmental policy

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given the regulator chose $X=ar{X}$

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The firm obtains profits:

$$\Pi=par{X}-C(ar{X})$$

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Let's do the comparative statics: differentiate the consumer's two FOCs with respect to \bar{X}

$$u_{XX} \frac{\partial \bar{X}}{\partial \bar{X}} + u_{XN} \frac{\partial N}{\partial \bar{X}} = \frac{\partial p}{\partial \bar{X}}$$
 (X FOC)

$$u_{NX} rac{\partial ar{X}}{\partial ar{X}} + u_{NN} rac{\partial N}{\partial ar{X}} = 0$$
 (N FOC)

 $\frac{\partial \bar{X}}{\partial \bar{X}} = 1$ so two equations, two unknowns;

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 $\frac{\partial X}{\partial \bar{X}} = 1$ so two equations, two unknowns; solving the system gives us:

$$egin{align} rac{\partial N}{\partial ar{X}} &= -rac{u_{XN}}{u_{NN}} \ rac{\partial p}{\partial ar{X}} &= rac{u_{XX}u_{NN} - u_{NN}^2}{u_{NN}} < 0 \ \end{pmatrix}$$

 $\operatorname{sign}(rac{\partial N}{\partial ar{X}})$ depends on whether X and N are complements or substitutes

Now that we know how the firm responds, return to the regulator's problem:

$$\max_{X,m} u(X,N) + Z - D(X) + pX - C(X) \quad ext{ s.t. } \quad wm(T-N) = G$$

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For convenience, we assume its returned to the consumer as a lump sum transfer so that:

$$Z=(1-m)w(T-N)-pX+G=(1-m)w(T-N)-pX+wm(T-N) \ \Rightarrow Z=w(T-N)-pX$$

Income is unchanged for a given level of N under the tax and transfer

The regulator's problem is then:

$$\max_{X,m} u(X,N) + \underbrace{w(T-N)}_Z - D(X) - C(X) + \lambda [wm(T-N) - G]$$

 λ is called the marginal welfare cost of public funds

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What's the FOC for m?

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Whats the interpretation of the right hand side?

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Why?

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Why?

Higher m increases leisure demand $\frac{\partial N}{\partial m}$

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Higher m increases leisure demand $\frac{\partial N}{\partial m}$

This times the tax wedge mw, the gap between w and actual wage after taxes, gives us the change in excess burden (i.e. the DWL d in the graph)

$$\lambda = rac{wmrac{\partial N}{\partial m}}{w(T-N)-wmrac{\partial N}{\partial m}}$$

The denominator is:

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First term is the increase in revenue on the inframarginal hours worked

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The denominator is:

The change in tax revenue from higher m

First term is the increase in revenue on the inframarginal hours worked

Second term is the decrease in revenue from reduced hours worked

• Similar to P(X) + P'(X)X for a monopolist

$$\lambda = rac{wmrac{\partial N}{\partial m}}{w(T-N)-wmrac{\partial N}{\partial m}}$$

Numerator and denominator combined give us:

The change in welfare from a change in m

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Numerator and denominator combined give us:

The change in welfare from a change in m over the change in tax revenue from a change in m

This is the change in welfare from a change in tax revenue!

Now consider the FOC for *X*:

$$[u_X-D'(X)-C'(X)+[u_N-w-\lambda wm]\,rac{\partial N}{\partial X}=0]$$

Now consider the FOC for *X*:

$$\left[u_X - D'(X) - C'(X) + \left[u_N - w - \lambda wm
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Recall that we know:

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So that we can substitute in the consumer leisure response:

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D'(X) is marginal damage

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What's the interpretation?

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It tells us how the optimal choice of X departs from X^* because of the labor market distortion

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Suppose *N* and *X* are substitutes, what does this mean?

Substitutes means that MIE > 0

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This exacerbates the distortion caused by the income tax: the household was already undersupplying labor because of the income tax

Now the household undersupplies labor to an even greater extent

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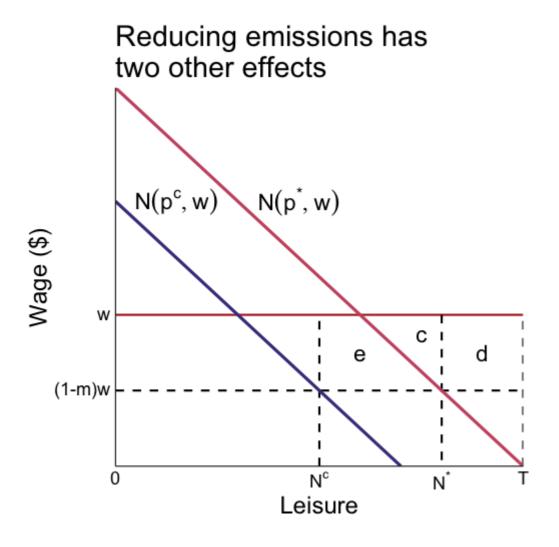
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Intuition?

Its less socially costly to reduce X because the household decreases N in response

This alleviates the distortion caused by the income tax: the household was undersupplying labor because of the income tax, but now reducing X increases labor supply, shrinking the labor market DWL



 $N^c
ightarrow N^*$ when $p^c
ightarrow p^*$ because of a change in X

This is
$$-\frac{\partial N}{\partial p} \frac{\partial p}{\partial X}$$

This reduces tax revenue by e+c which is just

$$egin{aligned} (N^*-N^c)(w-(1-m)w) \ &= \underbrace{(N^*-N^c)}_{pprox -rac{\partial N}{\partial p}rac{\partial p}{\partial X}} \end{aligned}$$

The marginal welfare cost of recovering the lost tax revenue (in order to maintain gov't revenues G) by raising m is λ giving us a total welfare cost of:

$$\lambda (N^*-N^c)mw$$

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But $(N^* - N^c)mw$ also happens to be the increase in excess burden: its a direct welfare loss in addition to the loss from having to increase m

So the total welfare loss is:

$$(1+\lambda)(N^*-N^c)mw$$

The discrete version of MIE!

Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

1. The marginal social cost of reducing X is higher if X and N are substitutes and lower if they are complements

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Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

- 1. The marginal social cost of reducing X is higher if X and N are substitutes and lower if they are complements
- 2. The optimal level of pollution is larger if they are substitutes, lower if they are complements
- 3. The absolute value of the difference in first and second-best pollution levels is larger if:
 - Demand for X is more inelastic
 - \circ Elasticity of substitution between N and X is greater

We didn't actually show the last part yet

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First define:

- ε_x as the own price elasticity $\frac{\partial X}{\partial p} \frac{p}{X}$
- η_{XN} as the elasticity of substitution between X and N: $\frac{\partial X}{\partial w} \frac{(1-m)w}{X}$

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and take advantage of the Slutsky symmetry condition $\partial N/\partial p = \partial X/\partial w$

We can then use these to substitute into the MIE and get:

$$MIE = (1 + \lambda) \left[-rac{\eta_{XN}}{arepsilon_X}
ight] p rac{m}{1-m}$$

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MIE bigger if $|\varepsilon_X|$ is smaller (more inelastic demand for X)

-->

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The regulator's problem is thus to select two tax rates: m and au

For simplicity we still assume all tax revenues are returned lump sum to households

First derive household spending on the numeraire good:

$$Z = (1-m)w(T-N) - pX + G = w(T-N) - pX + \tau X$$

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These are a function of the govt's choice of m and au

The household FOCs are:

$$u_X=p \qquad u_N=(1-m)w$$

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Next, as usual, differentiate the FOCs wrt τ

This gives us 3 equations and 3 unknown partial derivatives:

$$u_{XX} \frac{\partial X}{\partial \tau} + u_{XN} \frac{\partial N}{\partial \tau} = \frac{\partial p}{\partial \tau}$$
 (Household X FOC)

$$u_{XN} \frac{\partial X}{\partial au} + u_{NN} \frac{\partial N}{\partial au} = 0$$
 (N FOC)

$$C''(X)\frac{\partial X}{\partial \tau} = \frac{\partial p}{\partial \tau} - 1$$
 (Firm X FOC)

Substitute and solve...

Now solve for how the endogenous variables change in au

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$$\frac{\partial X}{\partial \tau} = \frac{u_{NN}}{H} < 0$$

$$rac{\partial N}{\partial au} = rac{-u_{XN}}{H} \lessgtr 0$$

$$rac{\partial p}{\partial au} = rac{u_{XX}u_{NN} - u_{XN}^2}{H} > 0$$

where
$$H=u_{XX}u_{NN}-u_{XN}^2-C^{\prime\prime}(X)u_{NN}>0$$

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The regulator wants to maximize social welfare given the budget constraint:

$$\max_{m, au} \underbrace{U(X,N) + Z - D(X)}_{ ext{household utility}} + \underbrace{pX - C(X) - au X}_{ ext{firm profit}}$$
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Substitute in for Z from household spending:

$$Z = w(T-N) - pX + au X$$

$$\left[u_X - C'(X) - D'(X)\right] \frac{\partial X}{\partial au} + \left[\underbrace{u_N - w}_{-wm} - \lambda wm\right] \frac{\partial N}{\partial au} + \lambda \left[X + au \frac{\partial X}{\partial au}\right] = 0$$

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Just follow the same steps as we did with the non-revenue raising case and divide by $\frac{\partial X}{\partial \tau}$ to get:

$$\underbrace{u_x - C'(X)}_{MAC} + \underbrace{(1+\lambda)wm \left[-\frac{\partial N}{\partial \tau} \middle/ \frac{\partial X}{\partial \tau} \right]}_{MIE} + \underbrace{\lambda \left[\tau + X \middle/ \frac{\partial X}{\partial \tau} \right]}_{MRE} = D'(X)$$

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Since the tax is per unit, we have that: $\frac{\partial N}{\partial \tau} / \frac{\partial X}{\partial \tau} = \frac{\partial N}{\partial p} / \frac{\partial X}{\partial p}$, MIE is similar in revenue and non-revenue raising contexts

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MRE changes the marginal social cost of X because changes in τ affect how much revenue we need to raise with distorting labor taxation

Let's get some intuition at the corner case of $\tau=0$

What's the sign of MRE?

$$MRE(au=0)$$
: $\lambda\left[x/rac{\partial X}{\partial au}
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- \rightarrow the additional revenue from an increase in τ lets us reduce labor taxes
- \rightarrow this reduces the distortionary tax in the labor market
- → this reduces welfare losses in the labor market
- \rightarrow this reduction in welfare losses reduces the marginal social cost of reducing X, decreasing the optimal level of X

Is MRE always negative?

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$$MRE \equiv \lambda \left[au + X \Big/ rac{\partial X}{\partial au}
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ight] = \lambda \left[au + p / arepsilon_X
ight] = \lambda au \left[1 + 1 / arepsilon_X^ au
ight]$$

where $\varepsilon_X < 0$ is the elasticity of demand for the dirty good and ε_X^{τ} is the elasticity with respect to the tax

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ight]$$

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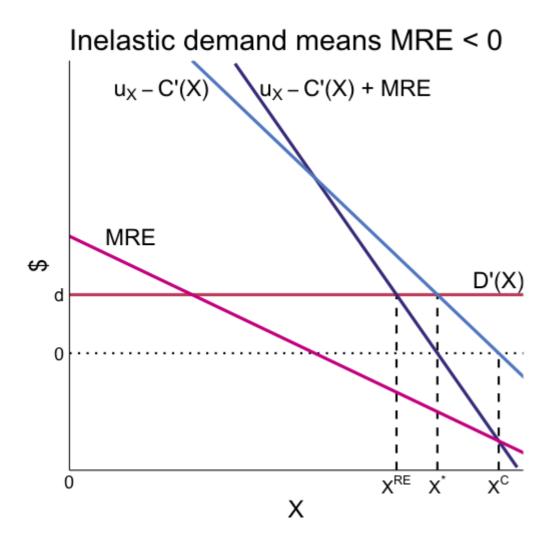
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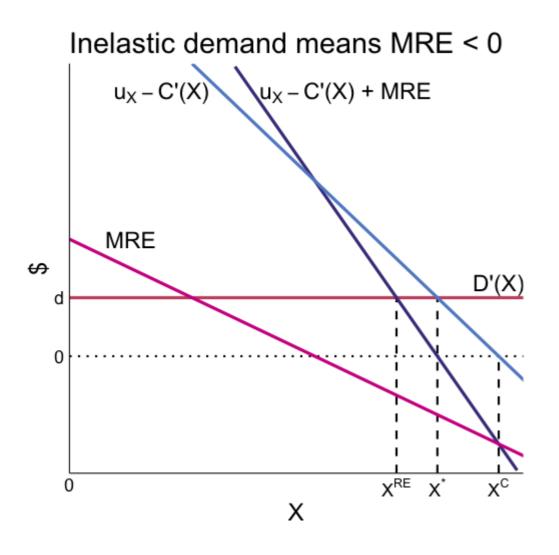
Why?



Demand for dirty good is sufficiently inelastic:

Suppose
$$rac{\partial N}{\partial p}=0$$
 so $MIE=0$, $C'(X)=c$, $D'(X)=d$

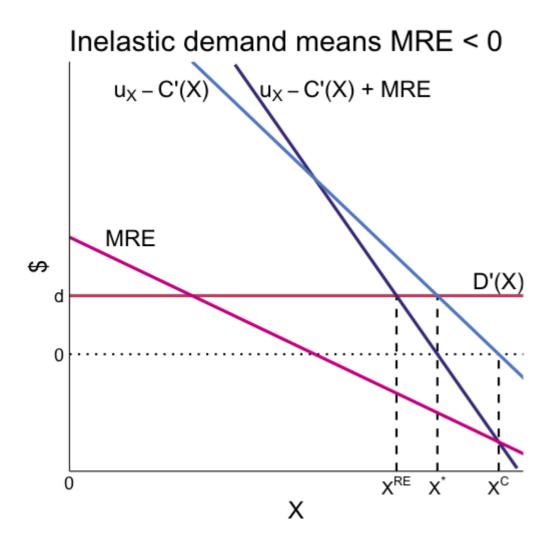
Inelastic demand lets us raise more revenue from a small change in the tax



Inelastic demand lets us raise more revenue from a small change in the tax

This reduces the marginal social cost of reducing X

Optimal X with revenue-raising is lower than without: $X^{RE} < X^*$



We can also see that if D'(X) was very large, making au larger, we would be where MRE>0

Is there a prospect for a double dividend?

There is a weak double dividend if welfare is always greater when revenue raised via environmental taxation is used to reduced distortionary taxation rather than refunded lump sum

This is always true

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There is a **strong double dividend** if the emission tax should always be set above the MAC=MD level, resulting in greater pollution reductions and more revenue raised

• This may or may not be true

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Price of X rises from τ , demand for leisure goes down, labor goes up

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Let's look at this pathway in more detail

Again, assume C'(X) = c, this gives us that:

$$MIE = \lambda \left(-rac{\eta_{XN}}{arepsilon_X}
ight) rac{p}{arepsilon_L} \qquad MRE = \lambda \left(rac{p}{arepsilon_X} + au
ight)$$

where

$$\eta_{XN} = \overbrace{\frac{\partial X}{\partial w} \frac{(1-m)w}{X}}^{\text{cross-price elasticity}} \qquad \varepsilon_L = \overbrace{-\frac{\partial N}{\partial w} \frac{(1-m)w}{L}}^{\text{labor supply elasticity}} = \frac{\partial L}{\partial w} \frac{(1-m)w}{L}$$

Suppose N and X are average substitutes which means the negative crossprice elasticity is equal to the labor supply elasticity $\eta_{XN} = \varepsilon_L$

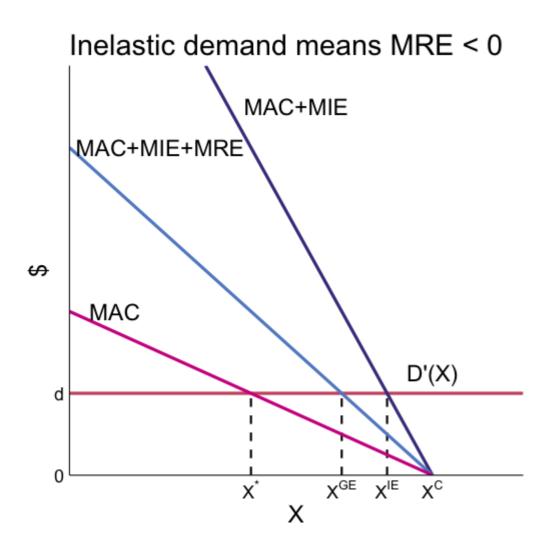
This is true if a 1% wage increase gives a $\eta_{XN}\% = \varepsilon_L\%$ spending increase

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$$MIE = \lambda \left(-rac{p}{arepsilon_X}
ight) < \lambda \left(rac{p}{arepsilon_X} + au
ight) = MRE$$

 \Rightarrow we shouldn't expect a strong double dividend because MIE + MRE = $\lambda au > 0$



Even though there isn't a double dividend, MIE and MRE still matter for the optimal second-best pollution level

Optimal pollution X^{GE} is larger than first-best X^* , but less than the level without revenue recycling X^{IE}

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What about freely allocated permits or command and control?

This would lead to the same *environmental* outcome, but not achieve the the welfare maximizing outcome

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Free allocation and command and control do not generate revenues that let us reduce labor taxation

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Setting $X^{GE} < X^c$ raises the price of X, increases leisure, and reduces revenues via the interaction effect

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Setting $X^{GE} < X^c$ raises the price of X, increases leisure, and reduces revenues via the interaction effect

Without revenue from permits or taxes, the optimal pollution level is higher