Lecture 6

Non-competitive output markets

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AEM 6510

Roadmap

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But markets are often **not** competitive

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Why might it?

Now we have **two** distortions in our market, pollution and market power

Environmental policy under monopoly

Suppose we have an industry with a **single** firm that generates a local pollutant

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We will again look at both the general and specific cases of our model

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Lets begin with the specific case where abatement is only possible through output reductions and $E = \delta X$

Environmental policy under monopoly

The monopolist's profit-maximization problem under an emission tax is:

$$\max_{X} \Pi(X) = P(X)X - C(X) - \tau E = P(X)X - C(X) - \tau\delta X$$

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Where the monopolist now controls X: aggregate production

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Where the monopolist now controls X: aggregate production

The FOC for the problem is:

$$P'(X)X + P(X) = C'(X) + \tau\delta$$

and the profit-maximizing output choice is given by $X^M(\tau)$

Environmental policy under monopoly

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This illustrates the monopolists decision rule: MR = MC

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MR consists of two pieces:

1. $P(X)$: Additional revenue from increased X
2. $P'(X)X$: decreases in the price for **inframarginal units** from increased X

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MR consists of two pieces:

1. $P(X)$: Additional revenue from increased X
2. $P'(X)X$: decreases in the price for **inframarginal units** from increased X

The monopolist now accounts for how increasing production lowers price

Environmental policy under monopoly

How does the firm respond to the tax? Differentiate the FOC wrt τ:

$$\left[P''(X^M)X^M + 2P'(X^M) - C''(X^M)\right]\frac{dX^M}{d\tau} = \delta$$

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and rearrange:

$$\frac{dX^M}{d\tau} = \frac{\delta}{[P''(X^M)X^M + 2P'(X^M) - C''(X^M)]}$$

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What's the sign on this expression?

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Let's make two assumptions

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$$\frac{dX^M}{d\tau} = \frac{\delta}{[P''(X^M)X^M + 2P'(X^M) - C''(X^M)]}$$

Assume the inverse demand function $P(X)$ is decreasing, and for all $X > 0$:

$$\frac{P''(X)}{P'(X)}X > -1$$

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This just comes from the second-order sufficient condition for a maximum being satisfied, it mean inverse demand isn't *too convex*: P'' is bounded above

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This just comes from the second-order sufficient condition for a maximum being satisfied, it mean inverse demand isn't *too convex*: P'' is bounded above

It also ensures that $\frac{dX^M}{d\tau} < 0$

Environmental policy under monopoly

Now lets look at the regulators problem of maximizing the benefits minus the costs of production, accounting for damages:

$$\max_X \int_0^X P(t)dt - C(X) - D(\delta X)$$

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She is maximizing the consumption value minus production and environmental costs

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She is maximizing the consumption value minus production and environmental costs

The FOC for this problem is:

$$P(X^*) = C'(X^*) + \delta D'(\delta X^*)$$

which doesn't map as nicely to the firm's FOC as in competitive markets

Environmental policy under monopoly

Lets re-write the regulator's problem and explicitly include the firm's optimal response to τ:

$$W(\tau) = \int_0^{X^m(\tau)} P(t)dt - C(X^m(\tau)) - D(\delta X^m(\tau))$$

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$$W(\tau) = \int_0^{X^m(\tau)} P(t)dt - C(X^m(\tau)) - D(\delta X^m(\tau))$$

Then differentiate with respect to τ to get:

$$[P(X^m) - C'(X^m) - \delta D'(\delta X^m)] \times \frac{dX^m}{d\tau} = 0$$

Environmental policy under monopoly

$$[P(X^m) - C'(X^m) - \delta D'(\delta X^m)] \times \frac{dX^m}{d\tau} = 0$$

Recognize $P(X^m) - C'(X^m) = \tau\delta - P'(X^m)X^M$ from the firm FOC

Environmental policy under monopoly

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Recognize $P(X^m) - C'(X^m) = \tau\delta - P'(X^m)X^M$ from the firm FOC

Rearranging gives us that the optimal tax rate is characterized by:

$$\tau = D'(\delta X^m) + \frac{P'(X^m)X^m}{\delta}$$

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Environmental policy under monopoly

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Can we achieve the first-best with this tax?

Yes! If we plug it into the firm FOC it is the same as the regulator's welfare-maximizing FOC

Environmental policy under monopoly

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What is the intuition behind this expression?

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The monopolist already reduces output to exercise its market power

Environmental policy under monopoly

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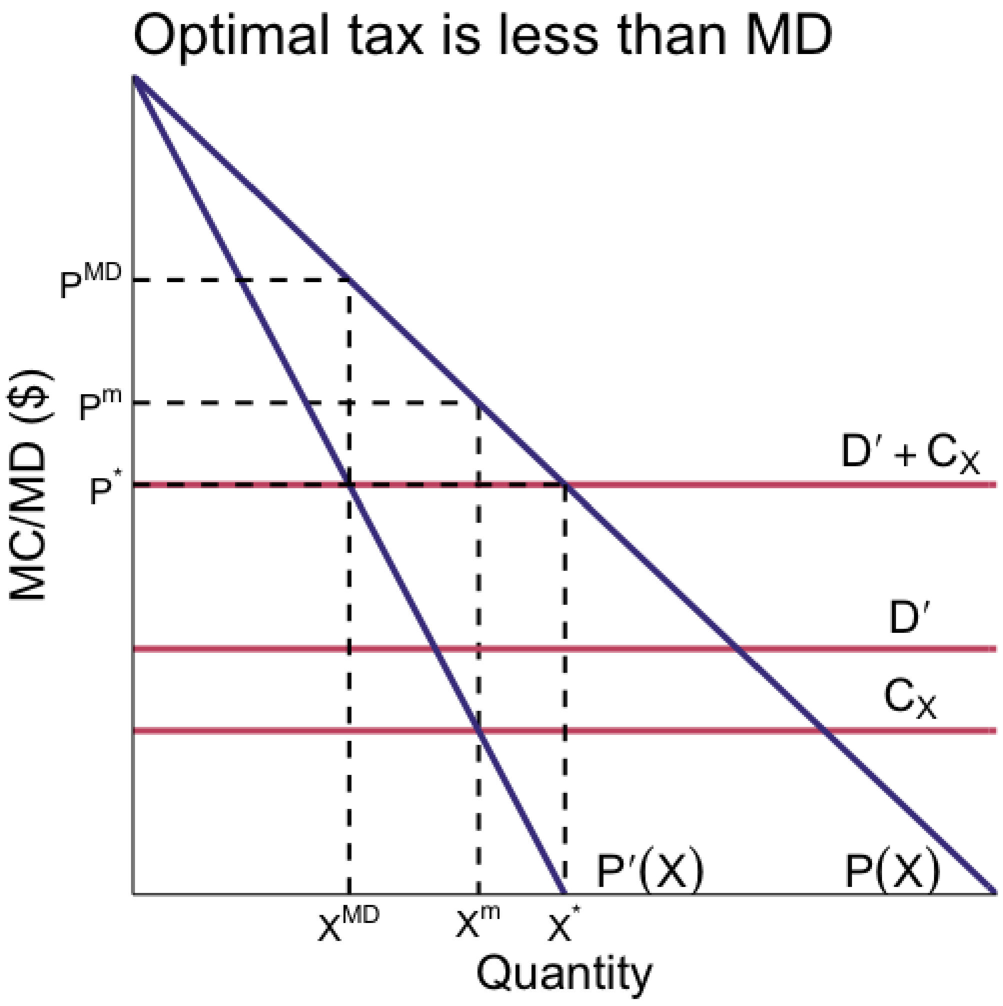
Why?

The monopolist already reduces output to exercise its market power

So we don't need as big of a tax, or may even need a **subsidy** if the monopolist was reducing output too much, to achieve the first-best

Graphical intuition for the optimal tax



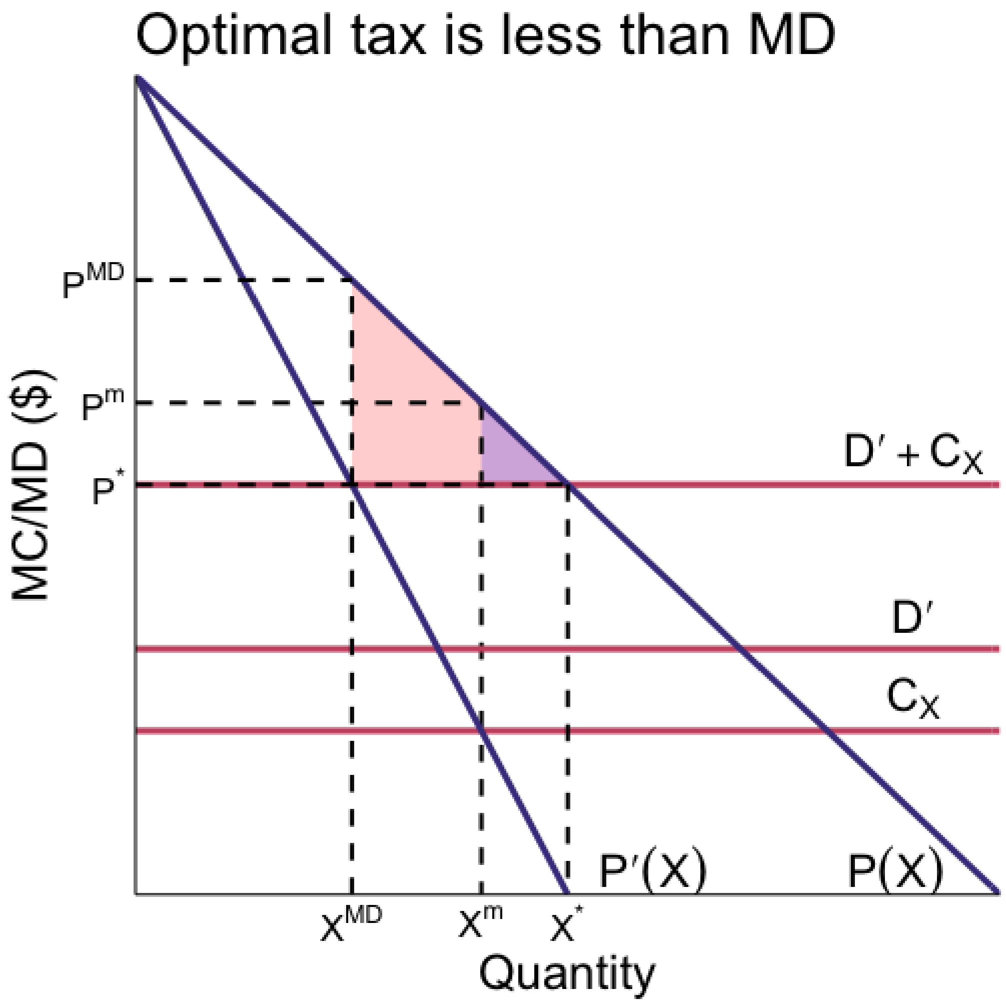
Suppose MD and MC are constant for simplicity and we are in the specific case

X^m, P^m is the unregulated monopoly allocation

X^{MD}, P^{MD} is the outcome if we set $\tau = D'(E^*)$

Since $X^{MD} < X^m < X^*$ this clearly **reduced welfare**

Graphical intuition for the optimal tax

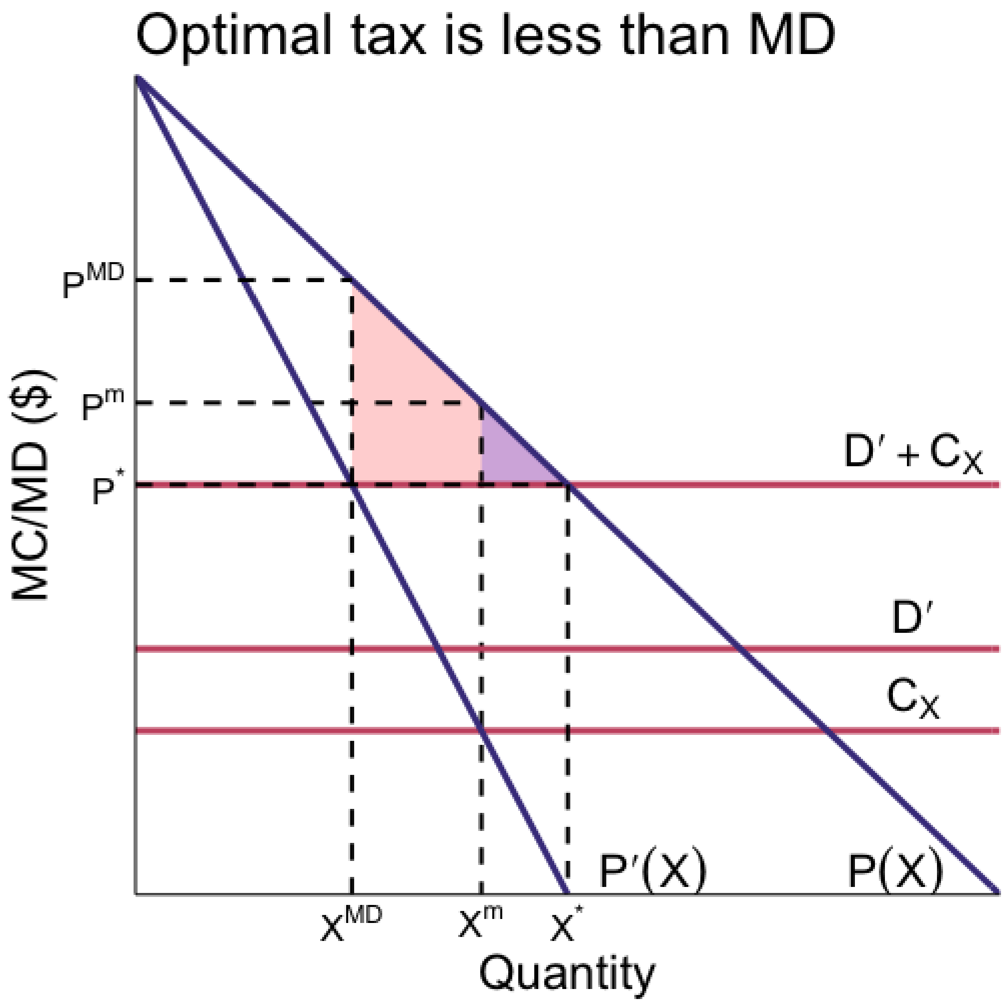


The purple/blue area is the original DWL under monopoly

The red + purple/blue area is the DWL if we tax at marginal damage, this **worsened welfare**

What's the optimal tax that gets us E^*?

Graphical intuition for the optimal tax



We want to shift the marginal cost of the firm so that it intersects MR at X^*, this vertical distance is the optimal tax

MR at X^* is 0, so we want to shift marginal cost down to zero

Our tax is then: $0 - C_X$: we actually **subsidize output**

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Does this hold more generally?

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Does this hold more generally?

The general monopoly problem is:

$$\max_{X,E} \Pi(X,E) = P(X)X - C(X,E) - \tau E$$

Environmental policy under monopoly

The FOCs are

$$P'(X)X + P(X) = C_X(X, E) \qquad - C_E(X, E) = \tau$$

The first is just MR = MC of production

The second is the MAC = tax for emissions

Environmental policy under monopoly

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Recall the regulators solution will look like:

$$P(X^*) = C_X(X^*, E^*) \qquad - C_E(X^*, E^*) = D'(E^*)$$

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Recall the regulators solution will look like:

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Can the regulator use an emission tax alone to achieve the efficient outcome?

Environmental policy under monopoly

The regulator's problem is:

$$W(\tau) = \int_0^{X^m(\tau)} P(t)dt - C(X^m(\tau), E^m(\tau)) - D(E^m(\tau))$$

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The FOC of this problem is:

$$[P(X^M) - C_X(X^m, E^m)]\frac{dX^m}{d\tau} - [C_E(X^m, E^m) + D'(E^m)]\frac{dE^m}{d\tau} = 0$$

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To get the **second-best** tax rate, substitute in the conditions from the firm FOCs:

$$P'(X)X + P(X) = C_X(X, E) \qquad -C_E(X, E) = \tau$$

Environmental policy under monopoly

This gives us that:

$$\tau = D'(E^m) + P'(X^m)X^m \frac{dX^m/d\tau}{dE^m/d\tau}$$

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What is the second term?

It represents how a change in emissions caused by the tax τ changes how the monopolist exercises market power

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It represents how a change in emissions caused by the tax τ changes how the monopolist exercises market power

$P'(X^m)X^m$ is the market power term in MR, $\frac{dX^m/d\tau}{dE^m/d\tau}$ is the output response to a tax-induced change in emissions

Environmental policy under monopoly

The sign of this term depends on $\frac{dX^m/d\tau}{dE^m/d\tau}$, so lets sign the two components

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We do this by differentiating the firm FOCs wrt τ and first solving for $dX^m/d\tau$:

$$P'(X)X + P(X) = C_X(X, E) \qquad - C_E(X, E) = \tau$$

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to give us that (after some algebra and dropping function arguments):

$$\frac{dX^m}{d\tau} = \frac{-C_{XE}}{C_{EE}\left[P''X + 2P' - C_{EE}\left\{C_{XX}C_{EE} - C_{XE}^2\right\}\right]} < 0$$

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We assumed $-C_{XE} > 0$, strict convexity ensures $C_{EE}, C_{XX}C_{EE} - C_{XE}^2 > 0$, and our most recent assumption that demand is not too convex ensures $P''X + 2P' < 0$

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The numerator is positive, the denominator is negative: output declines as the tax increases

Environmental policy under monopoly

For emissions we differentiate the firm FOCs wrt τ solve for $\frac{dE^m}{d\tau}$:

$$\frac{dE^m}{d\tau} = \frac{-1}{C_{EE}} + \frac{C_{XE}^2}{C_{EE}^2 \left[P''X + 2P' - C_{EE}\left\{C_{XX}C_{EE} - C_{XE}^2\right\}\right]} < 0$$

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As before, we assumed:

- $-C_{XE} > 0$
- Strict convexity ensures $C_{EE}, C_{XX}C_{EE} - C_{XE}^2 > 0$
- Demand is not too convex assumption ensures $P''X + 2P' < 0$

So both output and emissions decline in the tax rate

Environmental policy under monopoly

Can we use the tax alone to achieve the first-best?

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The last term doesn't equal zero so this FOC, given the second-best tax, cannot is not equal to the first-best condition

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The last term doesn't equal zero so this FOC, given the second-best tax, cannot is not equal to the first-best condition

The regulator cannot achieve the first-best in the general case with a tax alone

Environmental policy under monopoly

What ended up being the difference between the specific and general case?

Environmental policy under monopoly

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Environmental policy under monopoly

What ended up being the difference between the specific and general case?

In the specific case the monopolist only had one degree of freedom, output, and the regulator had one instrument, the tax

With one instrument and one degree of freedom the regulator can incentivize the firm to select a specific value of output AND emissions

Environmental policy under monopoly

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You cannot use one instrument to pin down two values (similar to solving one equation for two unknowns)

Environmental policy under monopoly

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The regulator still has only one instrument

You cannot use one instrument to pin down two values (similar to solving one equation for two unknowns)

When you have fewer instruments than market distortions you are in a **second-best world**

Environmental policy under monopoly

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If we tax equal to marginal damage, we will make the market power distortion worse

The second term in the tax expression reduces the tax to account for these concerns

Environmental policy under monopoly

We can achieve the first-best if we have two instruments to address two distortions

- Use an output subsidy to incentivize the monopolist to produce the efficient level
- Use an emission tax to get the efficient level of emissions

The monopolist's problem is:

$$\Pi(X,E) = [P(X) + \xi]X - C(X,E) - \tau E$$

Environmental policy under monopoly

The FOCs are:

$$P(X) + \xi + P'(X)X = C_X(X,E) \qquad - C_E(X,E) = \tau$$

If we set:

$$\xi = -P'(X^*)X^* \qquad \tau = D'(E^*)$$

the firm FOCs reduce to the regulator's efficiency conditions

Environmental policy under monopoly

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This is a special case of the *Tinbergen Rule* that says you need as many instruments as distortions to achieve the first-best