# Lecture 5

## Competitive output markets

Ivan Rudik
AEM 6510

# Roadmap

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But firms actually have production costs in addition to abatement costs, and sometimes these costs cannot be disentangled

Now we explore models where output and abatement may not be separable

This captures a wider range of potential abatement methods and technologies

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A firm's production technology is given by:

$$x = f(l_1, \ldots, l_K)$$

where $x$ is how many units of output are produced using production function $f$ and vector of inputs $\{l_1, \ldots, l_K\}$

# The competitive output model

A firm's emission technology is given by:

$$e = g(l_1, \ldots, l_K)$$

where $e$ is how many units of emissions resulting from using the vector of inputs $\{l_1, \ldots, l_K\}$

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Why?

An input that reduces emissions could reduce output, output-enhancing inputs could increase emissions or be emission-neutral

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We want to minimize the costs of producing a given combination of output $(x, e)$

$$C(x,e) = \min_{l_1,...,l_k} \left\{ \sum_{k=1}^{K} w_k l_k + \lambda \underbrace{[x - f(l_1, \ldots, l_K)]}_{\text{x units of output}} + \mu \underbrace{[e - g(l_1, \ldots, l_K)]}_{\text{e units of emissions}} \right\}$$

where $\{w_1, \ldots, w_K\}$ is a vector of input prices

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3. $C_{xe} = C_{ex} < 0 \;\; \forall e < \hat{e}^x$
4. $C_{xx} > 0, C_{ee} > 0, C_{xx}C_{ee} - C_{ex}^2 > 0$

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What do these tell us?

1. MC of production is positive and there is a cost-minimizing emission level $\hat{e}^x$ for all $x$ in the absence of environmental regulation

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3. MC of production is smaller with higher emissions $\leftrightarrow$ MAC shifts up if output rises
4. Production and abatement costs are convex (marginal costs are increasing)

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Differentiate $C_e(x, \hat{e}^x) = 0$ wrt $x$ to get: $C_{ex}(x, \hat{e}^x)\frac{dx}{dx} + C_{ee}(x, \hat{e}^x)\frac{d\hat{e}^x}{dx} = 0$

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Rearrange to get: $\frac{d\hat{e}^x}{dx} = -\frac{C_{ex}}{C_{ee}} > 0$

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- Emissions are given by $e = \delta(x)$ where $\delta'(x) > 0$
- In some cases we will assume $\delta(x) = \delta \cdot x$ to simplify

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If $p$ is the output price of $x$, profit is:

$$\Pi = px - c(x)$$

and if $e = \delta x$ we have that:

$$\frac{d\Pi}{de} = \frac{p - C'(e/\delta)}{\delta}$$

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**The MC of abatement is the forgone marginal profits from reducing emissions**

We can also see that the MAC is increasing:

$$\frac{d^2\Pi}{de^2} = \frac{C''(e/\delta)}{\delta^2} \geq 0$$

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Let consumer utility be:

$$U^i = U_i(x_i) + z_i - D_i(E)$$

where

- $x_i$ is the person's consumption level
- $z_i$ is spending on all other non- $x$ goods
- $D_i(E)$ are damages from aggregate emissions $E$
- There are $i = 1, \ldots, J$ consumers

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The consumer has a budget constraint:

$$y = px_i + z_i$$

where the price of $z_i$ is normalized to 1, $p$ is the price of $x$ in terms of $z$

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We can then derive gross benefits from consumption: $\int_0^{x_i} u_i'(s)ds = u_i(x_i)$

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Let $X = \sum_{i=1}^{I} x_i$ be aggregate consumption, $P(X)$ be the market inverse demand curve, and $D(E)$ be the aggregate damage curve

- You get $P(X)$ by just horizontally summing $p_i(x_i)$ like we did in previous classes

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$P(X)$ and $D(E)$ allow us to **fully** characterize benefits and damages to households

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Social welfare in the **general case** is given by:

$$W(x_1, \ldots, x_J, e_1, \ldots, e_J) \equiv \int_0^{X \equiv \sum x_j} P(s)ds - \sum_{j=1}^{J} C^j(x_j, e_j) - D(E)$$

where $j$ are specific firms, and household costs and firm revenues cancel out because they are just a transfer from households to firms

Welfare is CS minus total cost, minus damages

# The competitive output model

Social welfare in the specific case when $e_j = \delta_j(x_j)$ is given by:

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where $E = \sum_{j=1}^{J} \delta_j(x_j)$

Now we can derive the efficiency conditions for our model to understand what defines the optimal allocation

# The competitive output model: Efficiency

Begin with the **general case**, the FOCs are defined by:

$$\frac{\partial W}{\partial x_j} = P(X) - C^j_{x_j}(x_j, e_j) = 0 \rightarrow P(X) = C^j_{x_j}(x_j, e_j)$$

where $\frac{\partial}{\partial x_j} \int_0^{X \equiv \sum x_j} P(s)ds = P(X) \frac{\partial X}{\partial x_j} = P(X)$ by the fundamental theorem of calculus,

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$$\frac{\partial W}{\partial e_j} = C^j_{e_j}(x_j, e_j) + D'(E) = 0 \to D'(E) = -C^j_{e_j}(x_j, e_j)$$

These $2J$ equations give us the solutions $x^*_j, e^*_j$ for $j = 1, \ldots, J$

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For efficiency, we need to balance the environmental and production costs of producing the good with the benefits of consuming it

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Here only the $x_j$s are choice variables so we get the following FOCs:

$$P(X) = C_j'(x_j) + D'(E)\delta_j'(x_j) \quad j = 1, \ldots, J$$

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The left hand side is the marginal benefit of consumption

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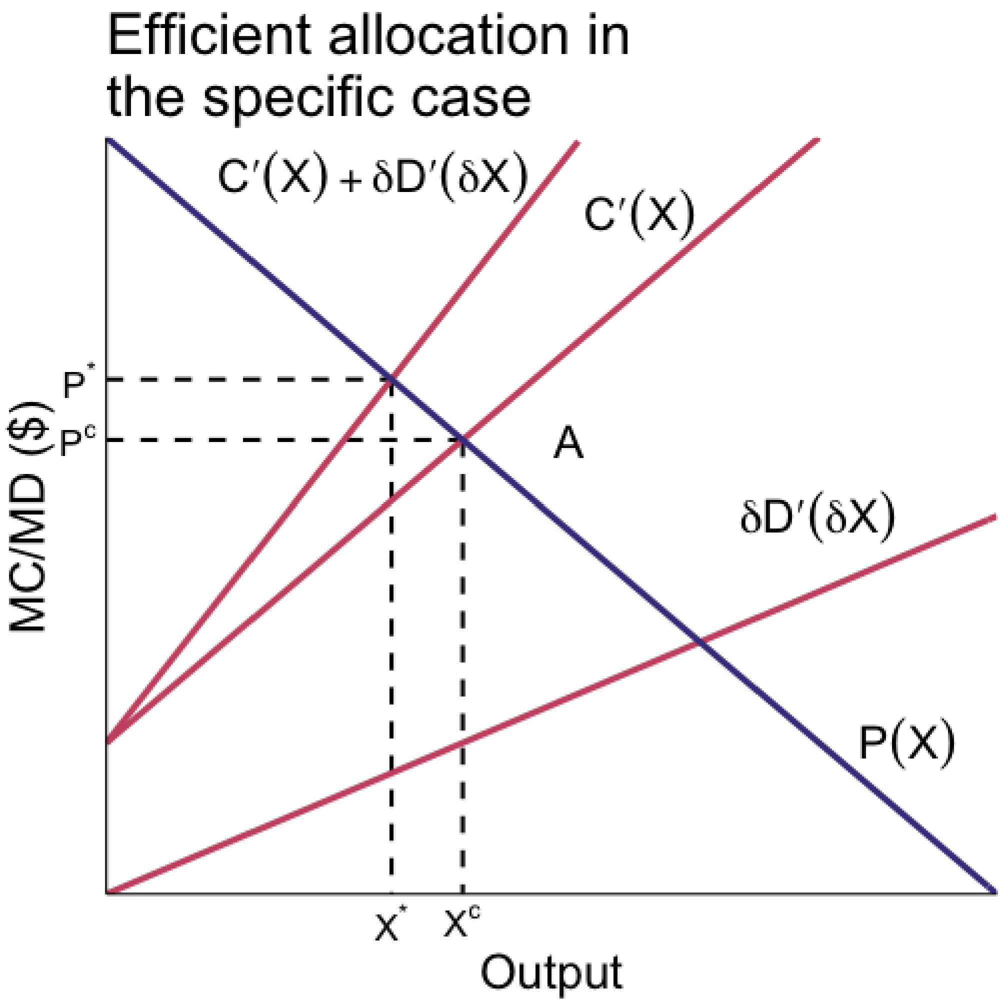
The left hand side is the marginal benefit of consumption

The right hand side is the total marginal cost:

- Private production costs
- External damage costs

# Efficiency in the specific case



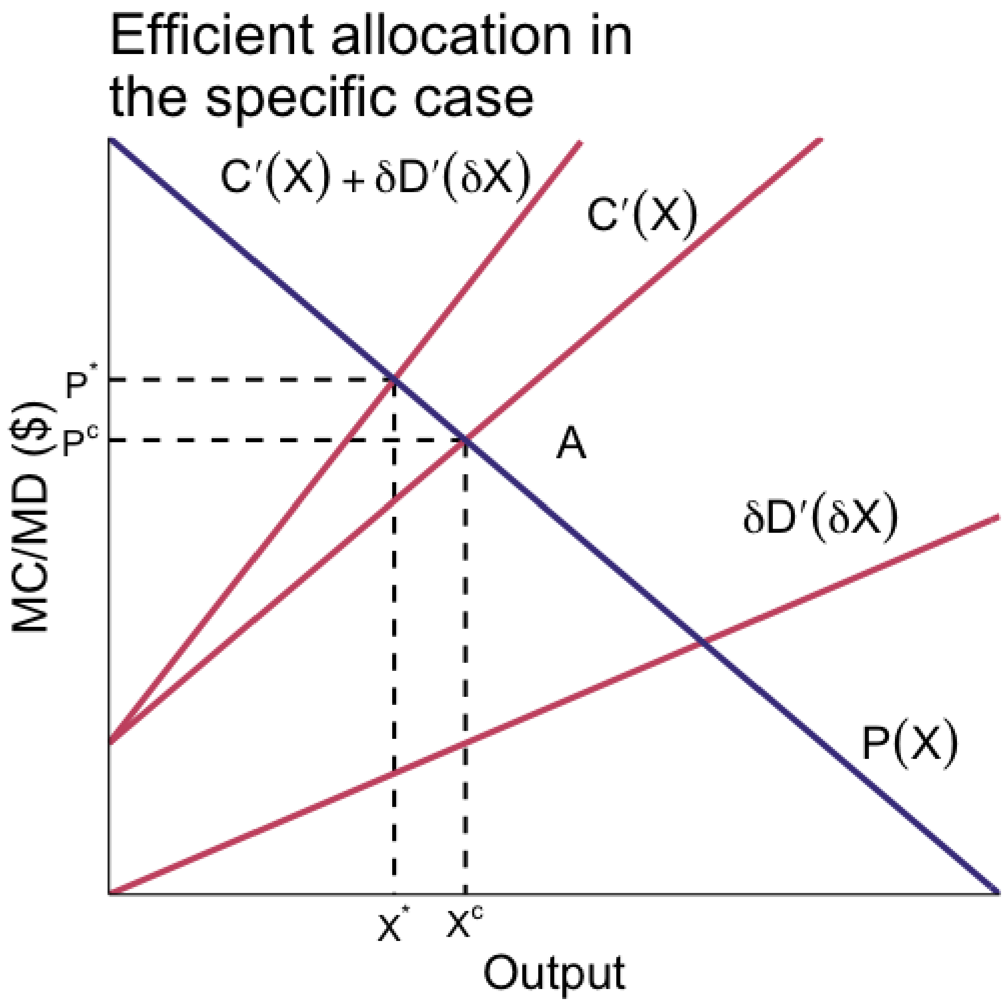

$X^c$ is the competitive allocation, this results in:

- Too much production
- Too low of an output price

$X^*$ is the optimal allocation where SMB = SMC

- This results in less production than the competitive allocation at a higher price

# Efficiency in the specific case




Where do the aggregate curves come from?

We get aggregate private MC $C'(X) = \sum_{j=1}^{J} C_j'(x_j)$ by *horizontally* summing firm MCs

We get SMC by *vertically* summing PMC and MD

# Policy instruments

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We will see that:

- There are additional results related to the output market that we didn't have before

- The previous results all still hold: taxes, subsidies, permits can all achieve the efficient allocation

# Policy instruments: taxes

When facing an emission tax $\tau$ a competitive firm's problem in the **general case** is:

$$\Pi_j(x_j, e_j) = px_j - C^j(x_j, e_j) - \tau e_j$$

where $p$ is the market price of $x$

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The firm equates the marginal abatement cost to the tax level

So if the regulator sets $\tau = D'(E^*)$ **we can achieve the efficient allocation**

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When facing an emission tax $\tau$ a competitive firm's problem in the specific case where $e_j = dx_j$ is:

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with FOC:

$$p = C_j'(x_j) + \tau\delta$$

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with FOC:

$$p = C_j'(x_j) + \tau\delta$$

If the regulator sets $\tau = D'(E^*)$ then firms behave as if

$$P(X) = C_j'(x_j) + \delta D'(E^*)$$

**which matches our social efficiency condition**

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To start we will assume all firms are identical and we are in the specific case of the model so our profit-maximizing firm FOC is:

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where $X = x \cdot J$

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where $X = x \cdot J$

Differentiate the FOC with respect to $\tau$ to get how output and emissions respond to a change in the tax rate

# Comparative statics

Differentiating gives us:

$$\left[P'(X)J - C''(x)\right] \frac{dx}{d\tau} = \delta$$

which implies that:

$$\frac{dx}{d\tau} = \frac{\delta}{P'(X)J - C''(x)} < 0$$

and $\frac{dX}{d\tau} = J\frac{dx}{d\tau} < 0$

Emission taxes reduce output

# Comparative statics

With $E = \delta \cdot X$ we have how emissions respond to the tax:

$$\frac{dE}{d\tau} = J\delta \frac{\delta}{P'(X)J - C''(x)} < 0$$

and since $p = P(X)$ is the market price of output, we can determine the relationship between output prices and the tax:

$$\frac{dp}{d\tau} = P'(X)\frac{dX}{d\tau} > 0$$

# Comparative statics

Recap: What do the comparative statics tell us?

Output and emissions decline in the tax:

- A tax on emissions raises the marginal cost of production for firms
  - Supply shifts left
- Output price $p$ goes up, quantity $x$ goes down

# Comparative statics

The **incidence** of the tax is also made clear by:

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Incidence is how the tax burden is distributed between consumers and producers

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The more the price of $x$ increases in response to a tax, the more the consumers pay for $x$ because of $\tau$, the higher their tax incidence

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Incidence is how the tax burden is distributed between consumers and producers

The more the price of $x$ increases in response to a tax, the more the consumers pay for $x$ because of $\tau$, the higher their tax incidence

Recall from Econ 101 that it doesn't matter who is taxed, the burden is shared by the consumers and producers

# Comparative statics

$$\frac{dp}{d\tau} = P'(X)\frac{dX}{d\tau} > 0$$

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If demand is perfectly elastic $P'(X) = 0$ and there is no associated price increase from a tax increase

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If demand is perfectly inelastic, then consumers bear the entire cost of the tax

# Comparative statics: taxes recap

What did we learn?

Increasing a tax:

1. Decreases firm and aggregate emission levels
2. Decreases firm and aggregate output (even in the general case, see pg 103-104 in the book)
3. Increases output prices

# Policy instruments: permits

Now suppose the regulator issues $L = E^*$ permits instead of setting a tax

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The regulator can achieve the first-best efficient outcome

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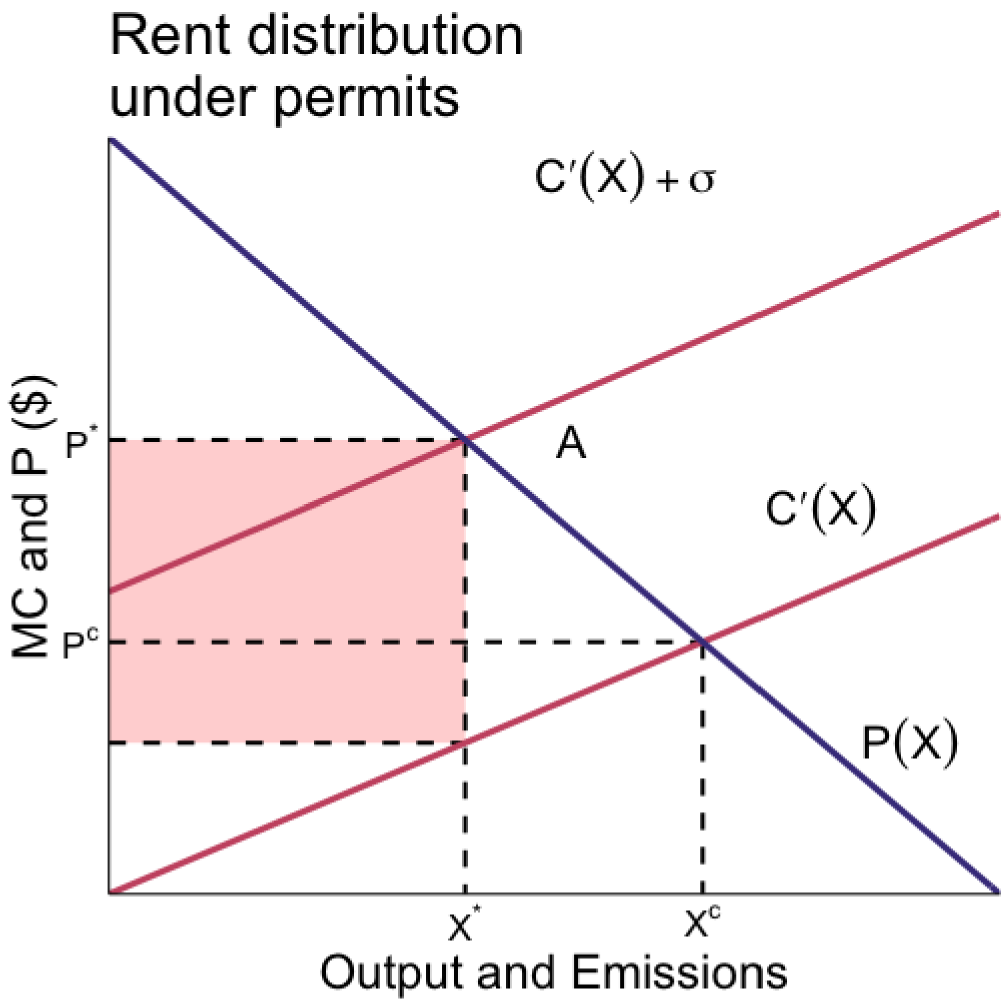
Output prices will also be the same because all firm and consumer decisions will be identical

The one way they will be different is how the **rents (economic profits)** are distributed: who gets the value from the scarcity of permits, the firms or the government?

# Distribution of rents in permit markets




Assume $x = \delta e$ and $\delta = 1$ so we can plot them on the same scale

The red shaded area is the rents from the permit scheme

If freely allocated: they remain with firms

If auctioned: they go to the government as revenue

# Relative/intensity standards

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or equivalently: $e \leq \alpha x$ where $\alpha$ is the policy variable

Relative standards are often called intensity standards because $e/x$ measures the pollution intensity of output

# Relative/intensity standards

Relative standards are only interesting in the **general case** of our model, in the specific case:

- If $\delta > \alpha$, the firm has to shut down
- If $\delta \leq \alpha$, the firm complies with the regulation no matter its actions

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- If $\delta > \alpha$, the firm has to shut down
- If $\delta \leq \alpha$, the firm complies with the regulation no matter its actions

Assume the regulation is binding in the **general case** (i.e. it actually affects firm behavior), then the firm will always set $e = \alpha x$[1]

This lets us re-write a firm's profit function as: $\Pi(x) = px - C(x, \alpha x)$

[1] Choosing $e$ strictly less than $\alpha x$ strictly raises costs and reduces profits.

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The FOC is:

$$p = C_x(x, \alpha x) + \alpha C_e(x, \alpha x)$$

The implicit solution to this, $x(\alpha)$, is the firms supply function, dependent on the policy $\alpha$

# Relative standards vs quantity standards

Suppose the regulator wants to hit $e = \bar{e}$ and she knows $x(\alpha)$

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What does this mean?

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**Total cost and marginal production and abatement costs are higher under a relative standard**

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Takeaways:

If a regulator sets an emission goal of $\bar{e}$ for a single firm or $J$ firms, then a relative standard will lead to:

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Why?

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Relative standards allow the firm to meet the standard *in ways we don't want them to*, so we end up with too much output

This limits us to **second-best** outcomes

# Relative standards: optimal policy

If we need to use a relative standard due to political or technical reasons what standard should we set to maximize welfare?

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The FOC is:

$$[P - C_x - \alpha C_e]x'(\alpha) - C_e x - D' \times [x + \alpha x'(\alpha)] = 0$$

where the term in the first bracket is 0 from the firm's $\pi$-max FOCs

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$x'(\alpha)$ tells us how output responds to policy and its sign tells us whether MAC $>$ MD or vice versa

Suppose $x'(\alpha) > 0$, then the second best policy sets MAC $>$ MD:

- The second-best quantity of emissions is lower than the first-best / optimal level that sets MAC = MD

# Environmental policy in the long run

So far we have assumed a fixed number of firms $J$

In the long run, firms may enter or exit the market, and this may affect the efficiency properties of our policies

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We will now look at long run properties of policy with identical firms

# Environmental policy in the long run

In the long run, firms have fixed and variable costs:

$$C(x, e) = \begin{cases} VC(x, e) + F \text{ if } x, e \neq 0 \\ 0 \text{ if } x, e = 0 \end{cases}$$

where $F$ is the fixed cost of entry, and $VC$ denotes variable costs of operation

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In the long run, social welfare will depend on $x, e$ **and the number of firms $J$**

# Environmental policy in the long run

Social welfare is given by:

$$W(x, e, J) = \max_{x,e,J} \int_0^{x \cdot J} P(t)dt - J \cdot C(x, e) - D(e \cdot J)$$

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The FOCs for a social optimum are:

$$\begin{aligned} P(X^*) &= C_x(x^*, e^*) && \text{(x FOC)} \\ D'(E^*) &= -C_e(x^*, e^*) && \text{(e FOC)} \\ P(X^*)x^* &= C(x^*, e^*) + D'(E^*)e^* && \text{(J FOC)} \end{aligned}$$

The last expression is the new one for long-run efficiency

# Environmental policy in the long run

With some slight rearranging of the $J$ FOC we can get:

$$P(X^*) = \frac{C(x^*, e^*) + D'(E^*)e^*}{x^*}$$

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With some slight rearranging of the $J$ FOC we can get:

$$P(X^*) = \underbrace{\frac{C(x^*, e^*) + \delta e^*}{x^*}}_{\text{average social cost}}$$

This means that it is socially efficient for firms to enter or exit until the price of output (approximately) equals the **average social cost curve**

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Firms can now only choose $x$ since $e$ is fixed at $e^*$

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MR = MC maximizes firm profit, zero profits ensures no change in # of firms

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When we impose a standard:

1. Firms cut back output
2. This raises (short-run) profit above zero
3. Firms enter the market until profits go to zero: so $\hat{J} > J^*$ and we will have that $\hat{x} < x^*$

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In the long run: standards do not appropriate the damage to the environment $D'(E^*)e^*$ from firms, so we get **excess entry** and standards are no longer first-best

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Giving us FOCs:

$$p = C_x(x^*, e^*) \qquad -C_e(x^*, e^*) = \tau$$

# Taxes in the long run

In the long run firms enter until profits are zero so:

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so $\tau = D'(E^*)$ implies that

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The firm FOCs for production and the entry zero-profit condition map exactly to the social welfare maximizing conditions!

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The firm FOCs for production and the entry zero-profit condition map exactly to the social welfare maximizing conditions!

The payment of tax rents from the firms to the regulator of $\tau e^* = D'(E^*)e^*$ limits entry and is what makes taxes efficient in the long run

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Suppose the regulator auctions off $L = E^* = e^* J$ permits and let $\sigma$ be the market-clearing permit price

The long run equilibrium is defined by the two firm FOCs and the entry condition:

$$
\begin{aligned}
&P(x^* J) = C_x(x^*, e^*) \\
&\sigma = -C_e(x^*, e^*) \\
&P(x^* J)x^* - C(x, e) - \sigma e^* = 0
\end{aligned}
$$

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Now what if we freely distribute permits? What do you think?

# Permits in the long run

Similar to the short run we will have that $\sigma = -C_e(x^*, e^*) = D'(L) = D'(E^*)$

Thus the three long run efficiency conditions are satisfied again if we auction off the permits:

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It seems like it might not be long run efficient: firms are not paying the environmental rent, so zero profit and P = ASC might not occur

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Suppose we allocate $\bar{e}$ permits to the incumbent identical firms, profit for the incumbent firms given permits is then:

$$\Pi = px - C(x, e) - \sigma(e - \bar{e})$$

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Our P = ASC condition can't hold for all firms

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Our efficiency condition is now **new firms enter until profits are zero (P=ASC for entrants)**:

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What is this saying?

**Operating profits** of any firms in the market are zero, incumbents had long run profits **only** from their initial permit allocation

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This is just an application of the Coase theorem: the initial assignment of property rights to pollute does not matter for efficiency

# Subsidies in the long run

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Denote the reference level of emissions as $\hat{e}$, firm profits under a subsidy per unit $\xi$ are:

$$\Pi = px - C(x, e) - \xi(e - \hat{e})$$

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which implies that

$$P(X^*)x^* - C(x^*, e^*) - D'(E^*)e^* = -D'(E^*)\hat{e} < 0$$

Too many firms have entered!

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Permits do not have these problems because the payment was only to incumbent firms, not entrants

Incumbent firms are already in the market: giving them the rents from freely distributed permits does not lead to excess entry