Heteroskedasticity in Time Series Regressions

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Econometrics II

Heteroskedasticity in Time Series Regressions

- We discussed testing and correcting for heteroskedasticity for cross-sectional applications.
- Heteroskedasticity can also occur in time series regression models, and the presence of heteroskedasticity, while not causing bias or inconsistency, does invalidate the usual standard errors, t statistics, and F statistics. This is just as in the cross-sectional case.
- In time series regression applications, heteroskedasticity often receives little, if any, attention compared to the problem of serially correlated errors.
- But it may be useful to briefly cover some of the issues that arise in applying tests and corrections for heteroskedasticity in time series regressions.

Heteroskedasticity in Time Series Regressions

- Suppose that Assumptions TS.1'-TS.2'-TS.3' and TS.5' do hold, but TS.4' is not valid.
- TS.2' rules out misspecifications such as omitted variables and certain kinds of measurement error.
- TS.5' rules out serial correlation in the errors.
- TS.4' is not valid. Homoskedasticity assumption does not hold in this case.

Heteroskedasticity in Time Series Regressions

- As in the cross-sectional data analysis, OLS is still unbiased and consistent if there is heteroskedasticity in the error term (assuming that the other assumptions hold).
- The usual OLS standard errors, t statistics, and F statistics can be adjusted to allow for the presence of heteroskedasticity of unknown form using **heteroskedasticty-robust standard errors**.
- If the only assumption violated is the homoskedasticity assumption, valid inference is easily obtained in most econometric packages.

Breusch-Pagan Test: Heteroskedasticity in Time Series Regressions

- The heteroskedasticty tests we covered in Chapter 8 can be applied directly to time series regression.
- **Breusch-Pagan test:**
- The first step of this test is to estimate the model using OLS and obtain the residuals.
- Then, the test(auxiliary) regression is estimated. We simply regress the squared residuals on the explanatory variables in the original model.
- Finally, we test the overall significance of the test regression via F test or LM test.
- If the calculated test statistic is significant, the null hypothesis of homoskedasticity is rejected in favor of the alternative hypothesis of heteroskedasticty.
- If the errors of the test regression is serially correlated, Breusch-Pagan test is not valid.

White Test: Heteroskedasticity in Time Series Regressions

- We can also use **White test** for the same purpose.
- The first step of this test is to estimate the model using OLS and obtain the fitted values of the model.
- Then, the test(auxiliary) regression is estimated. We simply regress the squared residuals on the fitted values and squared fitted values of the original model.
- Finally, we test the overall significance of the test regression via F test or LM test.
- If the calculated test statistic is significant, the null hypothesis of homoskedasticity is rejected in favor of the alternative hypothesis of heteroskedasticity If the errors of the test regression is serially correlated, White test is not valid.

Heteroskedasticity in Time Series Regressions

- If heteroskedasticity is found in the u_t (and the u_t are not serially correlated), then the heteroskedasticity-robust test statistics can be used. An alternative is to use weighted least squares (WLS), as in the case of cross sectional regression.
- In recent years, economists have become interested in dynamic forms of heteroskedasticity.
- Especially, the volatility in financial time series through time has an autoregressive form.

Example: Heteroskedasticity in Time Series Regression, EMH

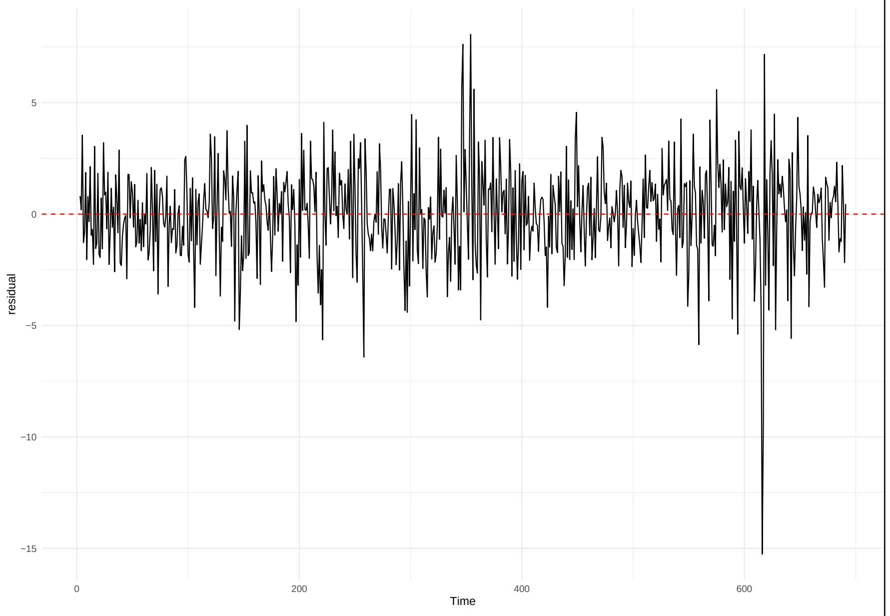
$$return_t = \beta_0 + \beta_1 return_{t-1} + u_t$$

- $t_{\beta_1} = 1.55$. it seems that there is no evidence against EMH !
- The test regression of Breusch-Pagan test:

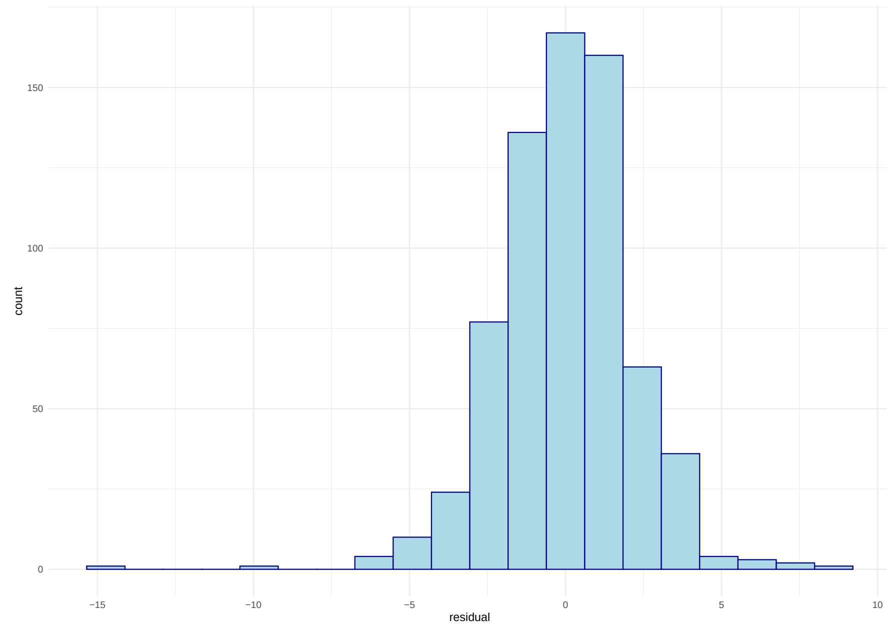
$$\widehat{u}_t^2 = \underset{(0.043)}{4.66} - \underset{(0.201)}{1.104}\, return_{t-1} + residual_t$$
$$n = 689 \quad R^2 = 0.042$$

- $t_{return_{t-1}} = -5.5$. Strong evidence in favor of heteroskedasticity.

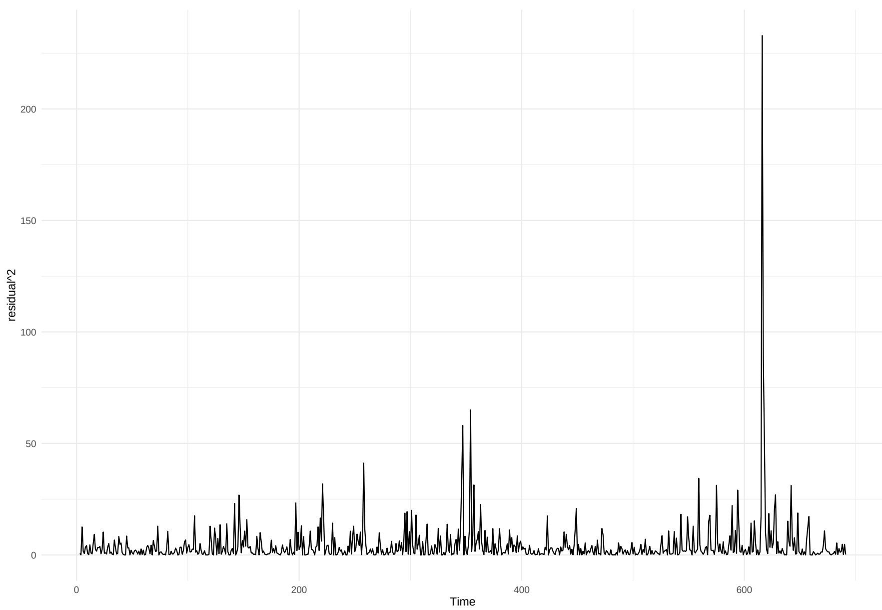
Plot of Residuals



Histogram of Residuals



Plot of Squared Residuals



Example: Heteroskedasticity in Time Series Regression, EMH

$$\hat{u}_t^2 = \underset{(0.043)}{4.66} - \underset{(0.201)}{1.104}\, return_{t-1} + residual_t$$

$$n = 689 \quad R^2 = 0.042$$

- we have the interesting finding that volatility in stock returns is lower when the previous return was high, and vice versa.
- Therefore, we have found what is common in many financial studies: the expected value of stock returns does not depend on past returns, but the variance of returns does.

Autoregressive Conditional Heteroskedasticity - ARCH

- Consider a simple static regression model:

$$y_t = \beta_0 + \beta_1 z_t + u_t$$

- Assume that the following model is valid for error terms:

$$E(u_t^2|u_{t-1}, u_{t-2}, ...) = E(u_t^2|u_{t-1}) = \alpha_0 + \alpha_1 u_{t-1}^2$$

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + v_t \quad ARCH(1) \quad Model$$

- This model makes sense if $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\alpha_1 < 1$.

Autoregressive Conditional Heteroskedasticity - ARCH

- If Gauss-Markov assumptions hold, OLS estimators are unbiased and consistent with ARCH errors. The usual statistical inference procedures are valid.
- If OLS still has desirable properties under ARCH, why should we care about ARCH type forms of heteroskedasticity in static and distributed lag models?
- First, it is possible to get consistent (but not unbiased) estimators that are asymptotically more efficient than the OLS estimators, e.g. weighted least squares (WLS)
- Second, in the ARCH model, larger magnitude of the error in the previous time period (u_{t-1}^2) was associated with a larger error variance in the current period. Since variance is often used to measure volatility, and volatility is a key element in asset pricing theories, ARCH models have become important in empirical finance.

Example: Autoregressive Conditional Heteroskedasticity - ARCH

- ARCH(1) in stock market returns

$$\hat{u}_t^2 = \underset{(0.44)}{+2.95} + \underset{(0.036)}{0.337}\,\hat{u}_{t-1}^2 + residual_t$$

$$n = 688 \quad R^2 = 0.114$$

- The t statistic on $\hat{u}_{t-1}^2$ is over nine, indicating strong ARCH. As we discussed earlier, a larger error at time $t-1$ implies a larger variance in stock returns today.
- It is important to see that, while the squared OLS residuals are autocorrelated, the OLS residuals themselves are not (as is consistent with the EMH).

Heteroskedasticity and Serial Correlation in Regression Models

- Both serial correlation and heteroskedasticity can be present in time series models.
- A viable option in practice is to model them together and correct them using a combined weighted least squares procedure. Consider the following model

$$y_t = \beta_0 + \beta_1 x_{t1} + \ldots + \beta_k x_{tk} + u_t$$
$$u_t = \sqrt{h_t}\nu_t$$
$$\nu_t = \rho\nu_{t-1} + e_t, |\rho| < 1$$

where the explanatory variables are independent of e_t for all t, and h_t is a function of x variables. The process e_t is serially uncorrelated and has a constant variance, σ_e^2.

Heteroskedasticity and Serial Correlation in Regression Models

- Under these assumptions, the error term u_t is both heteroskedastic and serially correlated,

$$\mathrm{Var}\left(u_t|\mathbf{x}_t\right) = \sigma_\nu^2 h_t$$

where $\sigma_\nu^2 = \sigma_e^2 / \left(1 - \rho^2\right)$

- Because $\nu_t = u_t/\sqrt{h_t}$ follows a stable AR(1) and is homoskedastic, we can write the transformed model as

$$y_t/\sqrt{h_t} = \beta_0(1/\sqrt{h_t}) + \beta_1\left(x_{t1}/\sqrt{h_t}\right) + \ldots + \beta_k\left(x_{tk}/\sqrt{h_t}\right) + \nu_t$$

- Because the error term follows an AR(1) process, we can apply the standard FGLS procedures, such as Prais-Winsten or Cohcrane-Orcutt.

Feasible GLS Estimation with AR(1) Serial Correlation and Heteroskedasticity

1. Estimate the regression model by OLS and save the residuals, $\hat{u}_t$
2. Regress $\log(\hat{u}_t^2)$ on $x_{t1}, x_{t2}, \ldots, x_{tk}$ and obtain the fitted values, say $\hat{g}_t$
3. Obtain the estimates of h_t: $\hat{h}_t = \exp(\hat{g}_t)$
4. Estimate the transformed equation:

$$\hat{h}_t^{-1/2} y_t = \hat{h}_t^{-1/2}\beta_0 + \beta_1 \hat{h}_t^{-1/2} x_{t1} + \ldots + \beta_k \hat{h}_t^{-1/2} x_{tk} + \text{error}_t$$

by standard Cochrane-Orcutt or Prais-Winsten methods