EC 607, Set 02

Edward Rubin Spring 2021

# Prologue

#### Last time

Research basics, our class, and R

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Material: The Rubin causal model (not mine), Chapter 2 MHE.

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#### **Future**

**Lab:** Meet Philip and start deepening R knowledge.

Long run: Deepen understandings/intuitions for causality and inference.

Research fundamentals

#### Research fundamentals

Angrist and Pischke provide four **fundamental questions for research**:

- 1. What is the **causal relationship of interest**?
- 2. How would an **ideal experiment** capture this causal effect of interest?
- 3. What is your **identification strategy**?
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Seemingly straightforward questions can be fundamentally unanswerable.

### General research recommendations

More unsolicited advice:

- Be curious.
- Ask questions.
- Attend seminars.
- Meet faculty (UO + visitors).
- Focus on learning—especially intuition.<sup>†</sup>
- Be kind and constructive.

<sup>†</sup> Learning is not always the same as getting good grades.

### What's so great about experiments?

Science widely regards experiments as the gold standard for research.

But why? The costs can be substantial.

#### **Costs**

- slow and expensive
- heavily regulated by (risk-averse?) review boards
- can abstract away from the actual question/setting

#### **Benefits**

So the benefits need to be pretty large, right?

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#### **Empirical exercise**

- 1. Collect data on health status and hospital visits.
- 2. Summarize health status by hospital-visit group.

### Example: Hospitals and health

Our empirical exercise from the 2005 National Health Inteview Survey:

Group	Sample Size	Mean Health Status	Std. Error
Hospital	7,774	3.21	0.014
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**Alternative conclusion:** Perhaps we're making a mistake in our analysis... maybe sick people go to hospitals?

### Potential outcomes framework

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- Binary treatment variable (e.g., hospitalized):  $\mathrm{D}_i=0,1$
- Outcome for individual i (e.g., health):  $Y_i$

This framework has a few names...

- Neyman potential outcomes framework
- Rubin causal model
- Neyman-Rubin "potential outcome" | "causal" "framework" | "model"

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The difference between these two outcomes gives us the **causal effect of hospital treatment**, *i.e.*,

$$\tau_i = \mathbf{Y}_{1i} - \mathbf{Y}_{0i}$$

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Accordingly, our methods try to address the related question

For each  $Y_{1i}$ , what is a (reasonably) good counterfactual?

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**Q** This comparison will return *an* answer, but is it *the* answer we want?

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**Selection bias** The extent to which the "control group" provides a bad counterfactual for the treated individuals.

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**Q** So how do experiments—the gold standard of empirical economic (and scientific) research—accomplish this goal and overcome selection bias?

### Back to experiments

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In other words: Randomly assigning  $D_i$  makes  $D_i$  independent of which outcome we observe (meaning  $Y_{1i}$  or  $Y_{0i}$ ).

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In other words, there is no selection bias, i.e.,

Selection bias 
$$= E[Y_{0i} \mid D_i = 1] - E[Y_{0i} \mid D_i = 0] = 0$$

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**A** Observational studies—comparing wage data from participants and non-participants—often find that people who complete these programs actually make lower wages.

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Governments subsidize training programs to assist disadvantaged workers.

**Q** Do these programs have the desired effects (*i.e.*, increase wages)?

A Observational studies—comparing wage data from participants and non-participants—often find that people who complete these programs actually make lower wages.

**Challenges** Participants self select. + Programs target lower-wage workers.

### Example: Training programs

How do we formalize these concerns in our framework?

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#### **Observational program evaluations**

$$E[\mathrm{Wage}_i \mid \mathrm{Program}_i = 1] - E[\mathrm{Wage}_i \mid \mathrm{Program}_i = 0] =$$

$$\underbrace{E[\mathrm{Wage}_{1i} \mid \mathrm{Program}_i = 1] - E[\mathrm{Wage}_{0i} \mid \mathrm{Program}_i = 1]}_{\text{Average causal effect of training program on wages for participants, } i.e., \bar{ au}_1 + \underbrace{E[\mathrm{Wage}_{0i} \mid \mathrm{Program}_i = 1] - E[\mathrm{Wage}_{0i} \mid \mathrm{Program}_i = 0]}_{\text{Selection bias}}$$

#### Example: Training programs

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#### Observational program evaluations

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 $E[\mathrm{Wage}_{0i} \mid \mathrm{Program}_i = 1] - E[\mathrm{Wage}_{0i} \mid \mathrm{Program}_i = 0]$ 
Selection bias

If the program attracts/selects individuals who, on average, have lower wages without the program (sort of the point of the program), then we have negative selection bias.

#### Example: Training programs

```
\begin{split} E[\text{Wage}_i \mid & \text{Program}_i = 1] - E[\text{Wage}_i \mid \text{Program}_i = 0] = \\ & E[\text{Wage}_{1i} \mid \text{Program}_i = 1] - E[\text{Wage}_{0i} \mid \text{Program}_i = 1] + \\ & E[\text{Wage}_{0i} \mid \text{Program}_i = 1] - E[\text{Wage}_{0i} \mid \text{Program}_i = 0] \end{split}
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So even if the program, on average, has an positive wage effect (in the participant group), i.e.,  $\bar{\tau}_1 > 0$ , we will detect a lower effect due to the negative selection bias.

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**Related** While observational studies typically found negative program effects, several experiments found positive program effects.

#### Example: The STAR experiment

The Tennessee STAR experiment is a famous/popular example of an experiment that allows us to answer an important social/policy question.

**Research question** Do classroom resources affect student performance?

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- Ran for 4 years with ~11,600 children. Cost ~\$12 million.

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**Research question** Do classroom resources affect student performance?

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#### **Treatments**

- 1. Small classes (13–17 students)
- 2. Regular classes (22–35 students) plus part-time teacher's aide
- 3. Regular classes (22–35 students) plus full-time teacher's aide

#### Example: The STAR experiment

**First question** Did the randomization balance participants' characteristics across the treatment groups?

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Ideally, we would have pre-experiment data on outcome variable.

Unfortunately, we only have a few demographic attributes.

Table 2.2.1, MHE

	Tı			
Variable	Small	Regular	Regular + Aide	P-value
Free lunch	0.47	0.48	0.50	0.09
White/Asian	0.68	0.67	0.66	0.26
Age in 1985	5.44	5.43	5.42	0.32
Attrition rate	0.49	0.52	0.53	0.02
K. class size	15.10	22.40	22.80	0.00
K. test percentile	54.70	48.90	50.00	0.00

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Demographics appear balanced across the three treatment groups.

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The three groups differ significantly on attrition rate.

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The randomization generated variation in the treatment.

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The small-class treatment significantly increased test scores.

#### The STAR experiment

The previous table estimated/compared the treatment effects using simple differences in means.

We can make the same comparisons using regressions.

Specifically, we regress our outcome (test percentile) on dummy variables (binary indicator variables) for each treatment group.

Example of our three treatment dummies.

i	$y_i$	$\mathrm{Trt}_{1i}$	$\mathrm{Trt}_{2i}$	$\mathrm{Trt}_{3i}$
1	$y_1$	1	0	0
2	$y_2$	1	0	0
•	• •	• •	:	•
$\ell$	$y_\ell$	1	0	0
$\ell+1$	$y_{\ell-1}$	0	1	0
•	•	•	•	•
p	$y_p$	0	1	0
p+1	$y_{p+1}$	0	0	1
•	•	•	•	•
N	$y_N$	0	0	1

### Regression analysis

Assume for the moment that the treatment effect is constant<sup>†</sup>, i.e.,

$$\mathbf{Y}_{1i} - \mathbf{Y}_{0i} = \boldsymbol{\rho} \quad \forall i$$

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$$\mathbf{Y}_i = \underbrace{lpha}_{=E[\mathbf{Y}_{0i}]} + \mathbf{D}_i \underbrace{oldsymbol{
ho}}_{\mathbf{Y}_{1i}-\mathbf{Y}_{0i}} + \underbrace{\eta_i}_{\mathbf{Y}_{0i}-E[\mathbf{Y}_{0i}]}$$

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$$Y_i = \alpha + D_i \rho + \eta_i$$

Now write out the conditional expectation of  $Y_i$  for both levels of  $D_i$ 

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Again, our estimate of the **treatment effect** ( $\rho$ ) is only going to be as good as our ability to shut down the **selection bias**.

Selection bias in regression model: 
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### Regression analysis

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There is something in our disturbance  $\eta_i$  that is affecting  $Y_i$  and is also correlated with  $D_i$ .

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In other metrics-y words: Our treatment  $D_i$  is endogenous.

#### Solutions and covariates

Selection bias in regression model:  $E[\eta_i | \mathbf{D}_i = 1] - E[\eta_i | \mathbf{D}_i = 0]$ 

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Another potential route to identification is to condition on covariates in the hopes that they "take care of" the relationship between  $D_i$  and whatever is in our disturbance  $\eta_i$ .

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Another potential route to identification is to condition on covariates in the hopes that they "take care of" the relationship between  $D_i$  and whatever is in our disturbance  $\eta_i$ .

Without very clear reasons explaining how you know you've controlled for the "bad variation", clean and convincing identification on this path is going to be challenging.

#### Covariates

That said, covariates can help with two things:

- 1. Even experiments may need **conditioning/controls**: The STAR experiment was random within school—not across schools.
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Now that we've seen regression can analyze experiments, let's estimate the STAR example...

Table 2.2.2, MHE

Explanatory variable	1	2	3
Small class	4.82	5.37	5.36
	(2.19)	(1.26)	(1.21)
Regular + aide	0.12	0.29	0.53
	(2.23)	(1.13)	(1.09)
White/Asian			8.35
			(1.35)
Female			4.48
			(0.63)
Free lunch			-13.15
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School F.E.	F	Т	Τ

The omitted level is Regular (with part-time aide).

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Results without other controls are very similar to the difference in means.

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School FEs enforce the experiment's design and increase precision.

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Additional controls slightly increase precision.

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