

Controls

EC 425/525, Set 6

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Prologue

Schedule

Last time

The conditional independence assumption: $\{Y_{0i}, Y_{1i}\} \perp\!\!\!\perp D_i | X_i$
i.e., conditional on some controls (X_i), treatment is as-good-as random.

Today

- Omitted variable bias
- Good vs. bad controls

Upcoming

- Topics: Matching estimators
- Admin: Assignment and midterm

Omitted-variable bias

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Revisiting an old friend

Let's start where we left off: Returns to schooling.

We have two linear, population models

$$Y_i = \alpha + \rho s_i + \eta_i \quad (1)$$

$$Y_i = \alpha + \rho s_i + \mathbf{X}'_i \boldsymbol{\gamma} + \nu_i \quad (2)$$

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For model (2), we can interpret $\hat{\rho}$ causally **if** $Y_{si} \perp\!\!\!\perp s_i | \mathbf{X}_i$ (CIA).

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For model (2), we can interpret $\hat{\rho}$ causally **if** $Y_{si} \perp\!\!\!\perp s_i | \mathbf{X}_i$ (CIA).

In other words, the CIA says that our **observable vector \mathbf{X}_i must explain all of correlation between s_i and η_i .**

Omitted-variable bias

The OVB formula

We can use the omitted-variable bias (OVB) formula to compare regression estimates from **models with different sets of control variables**.

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We're concerned about selection and want to use a set of control variables to account for ability (\mathbf{A}_i)—family background, motivation, intelligence.

$$Y_i = \alpha + \beta s_i + v_i \quad (1)$$

$$Y_i = \pi + \rho s_i + \mathbf{A}'_i \gamma + e_i \quad (2)$$

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What happens if we can't get data on \mathbf{A}_i and opt for (1)?

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What happens if we can't get data on \mathbf{A}_i and opt for (1)?

$$\frac{\text{Cov}(Y_i, s_i)}{\text{Var}(s_i)} = \rho + \gamma' \delta_{As}$$

where δ_{As} are coefficients from regressing \mathbf{A}_i on s_i .

Omitted-variable bias

Interpretation

Our two regressions

$$Y_i = \alpha + \beta s_i + v_i \quad (1)$$

$$Y_i = \pi + \rho s_i + \mathbf{A}'_i \gamma + e_i \quad (2)$$

will yield the same estimates for the returns to schooling

$$\frac{\text{Cov}(Y_i, s_i)}{\text{Var}(s_i)} = \rho + \gamma' \delta_{As}$$

if **(a)** schooling is uncorrelated with ability ($\delta_{As} = 0$) or **(b)** ability is uncorrelated with earnings, conditional on schooling ($\gamma = 0$).

Omitted-variable bias

Example

Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

Here we have four specifications of controls for a regression of log wages on years of schooling (from the NLSY).

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Column 1 (no control variables) suggests a 13.2% increase in wages for an additional year of schooling.

Omitted-variable bias

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Column 2 (age dummies) suggests a 13.1% increase in wages for an additional year of schooling.

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Schooling	0.132	0.131	0.114	0.087
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Column 3 (column 2 controls plus parents' ed. and self demographics) suggests a 11.4% increase in wages for an additional year of schooling.

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Column 4 (column 3 controls plus AFQT[†] score) suggests a 8.7% increase in wages for an additional year of schooling.

[†] AFQT is Armed Forces Qualification Test.

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As we ratchet up controls, the estimated returns to schooling drop by 4.5 percentage points (34% drop in the coefficient) from **Column 1** to **Column 4**.

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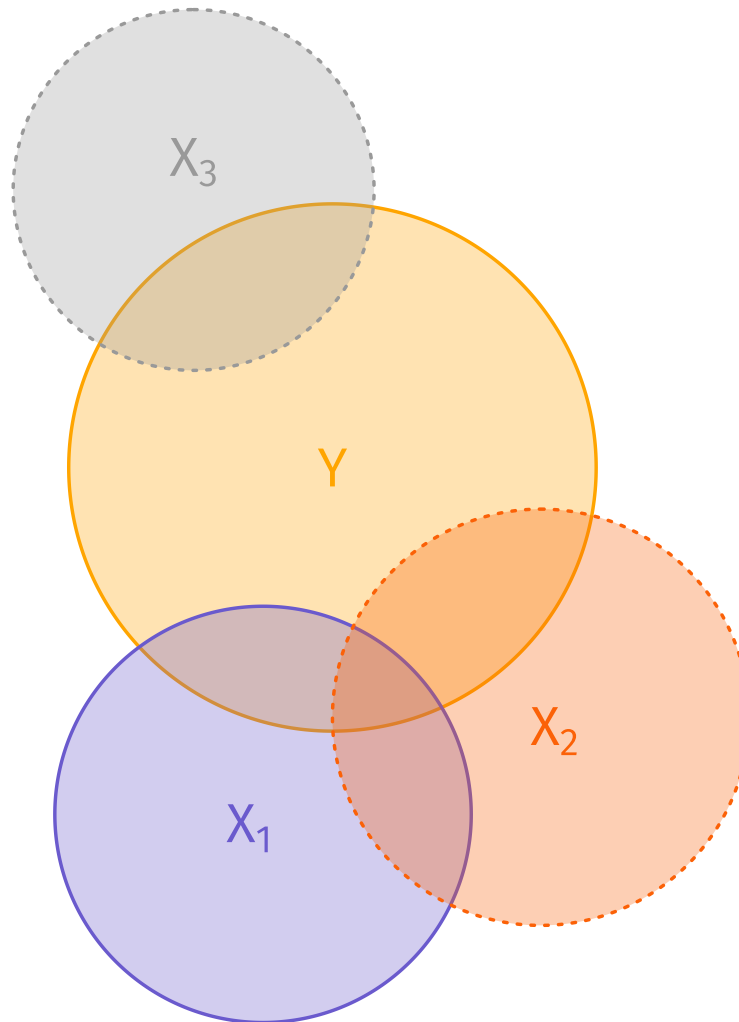
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$$\frac{\text{Cov}(Y_i, s_i)}{\text{Var}(s_i)} = \rho + \gamma' \delta_{As}$$

If we think **ability positively affects wages**, then it looks like we also have **positive selection into schooling**.

Omitted: X_2 and X_3



Omitted-variable bias

Note

This OVB formula **does not** require either of the models to be causal.

The formula compares the regression coefficient in a **short model** to the regression coefficient on the same variable in a **long model**.[†]

[†] Here, **long model** refers to a model with more controls than the **short model**.

Omitted-variable bias

The OVB formula and the CIA[†]

In addition to helping us think through and sign OVB, the formula

$$\frac{\text{Cov}(Y_i, s_i)}{\text{Var}(s_i)} = \rho + \gamma' \delta_{As}$$

drives home the point that we're leaning *very* hard on the conditional independence assumption to be able to interpret our coefficients as causal.

[†] The title for my first spy novel.

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Q When is the CIA plausible?

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Q When is the CIA plausible?

A Two potential answers

1. Randomized experiments
2. Programs with arbitrary cutoffs/lotteries

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Control variables play an enormous role in our quest for causality (the CIA).

Q Are "more controls" always better (or at least never worse)?

A No. There are such things as...

Bad controls

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Defined

Q What's a *bad* control—when can a control make a bad situation worse?

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Hint It's a flavor of selection bias.

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Hint It's a flavor of selection bias.

Let's consider an example...

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Example

Suppose we want to know the **effect of college graduation on wages**.

1. There are only two types of jobs: blue collar and white collar.
2. White-collar jobs, on average, pay more than blue-collar jobs.
3. Graduating college increases the likelihood of a white-collar job.

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A No. Imagine college degrees are randomly assigned. When we condition on occupation, we compare degree-earners who chose blue-collar jobs to non-degree-earners who chose blue-collar jobs. Our assumption of random degrees says **nothing** about random job selection.

Bad controls

Formal-ish derivation

More formally, let

- W_i be a dummy for whether i has a white-collar job
- Y_i denote i 's earnings
- C_i refer to i 's **randomly assigned** college-graduation status

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Because we've assumed C_i is randomly assigned, differences in means yield causal estimates, *i.e.*,

$$E[Y_i | C_i = 1] - E[Y_i | C_i = 0] = E[Y_{1i} - Y_{0i}]$$
$$E[W_i | C_i = 1] - E[W_i | C_i = 0] = E[W_{1i} - W_{0i}]$$

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Let's see what happens when we throw in some controls—*e.g.*, focusing on the the wage-effect of college graduation for white-collar jobs.

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Formal-ish derivation, continued

By introducing a bad control, we introduced selection bias into a setting that did not have selection bias without controls.

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Specifically, the selection bias term

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describes how college graduation changes the composition of the pool of white-class workers.

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Note Even if the causal effect is zero, this selection bias need not be zero.

Bad controls

A trickier example

A timely/trickier example: Wage gaps (*e.g.*, female-male or black-white).

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- What are we trying to capture?
- If we're concerned about discrimination, it seems likely that discrimination also affects occupational choice and hiring outcomes.
- Some motivate occupation controls with groups' differential preferences.

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- Some motivate occupation controls with groups' differential preferences.

What's the answer?

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Proxy variables

Angrist and Pischke bring up an interesting scenario that intersects omitted-variable bias and bad controls.

- We want to estimate the returns to education.
- Ability is omitted.
- We have a proxy for ability—a test taken after schooling finishes.

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We're a bit stuck.

1. If we omit the test altogether, we've got omitted-variable bias.
2. If we include our proxy, we've got a back control.

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1. If we omit the test altogether, we've got omitted-variable bias.
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With some math/luck, we can bound the true effect with these estimates.

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Example

Returning to our OVB-motivated example, we control for occupation.

Table 3.2.1, The returns to schooling

	1	2	3	4	5
Schooling	0.132	0.131	0.114	0.087	0.066
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Controls	None	Age Dum.	2 + Add'l	3 + AFQT	4 + Occupation

Schooling likely affects occupation; how do we interpret the new results?

Bad controls

Conclusion

Timing matters.

The right controls can help tremendously, but bad controls hurt.

Table of contents

Admin

1. Schedule

Controls

1. Omitted-variable bias
 - The formula
 - Example
 - OVB Venn
 - OVB and the CIA
2. Bad controls
 - Defined
 - Example
 - Formalization(ish)
 - Trickier example
 - Bad proxy conundrum
 - Empirical example