Instrumental Variables

EC 421, Set 11

Edward Rubin

Prologue

Schedule

Last Time

Causality

Today

- Review: Causality
- New: Instrumental variables

Causality

Review

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$$\tau_i = y_{1i} - y_{0i}$$

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Fundamental problem of causal inference:

We cannot simultaneously know y_{1i} and y_{0i}.

Causality

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or we observe i in the control group, *i.e.*,

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but never both at the same time.

Causality

Review

If we want to know τ_i (or at least $\bar{\tau}$), what can we do?

Causality

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Idea: Estimate the **average treatment effect** as the difference between the average outcomes in the treatment group and the control group, *i.e.*,

$$Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0)$$

where $D_i = 1$ if i received treatment, and $D_i = 0$ if i is in the control group.

Review

Result: We showed that even when the treatment effect is constant (meaning $\tau_i = \tau$ for all i),

$$\begin{aligned} & Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0) \\ & = \tau + \underbrace{Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)}_{\text{Selection bias}} \end{aligned}$$

which says that the difference in the groups' means will give us a **biased estimate** for the causal effect of treatment **if we have selection bias.**

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Causality

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Specifically, the control group doesn't give us a good **counterfactual** for what our treatment group would have looked like if the members had not received treatment.

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A: (Formal) The average *untreated* outcome for a member of our **treatment group** (which we cannot observe) differs from the average *untreated* outcome for a member of our **control group**, *i.e.*,

$$Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)$$

Causality

Review

Practical problem: Selection bias is also difficult to observe

$$\underbrace{Avg(y_{0,i} \mid D_i = 1)}_{\text{Unobservable}} - Avg(y_{0,i} \mid D_i = 0)$$

(back to the *fundamental problem of causal inference*)

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Bigger problem: If selection bias is present, our estimate for τ is biased, preventing us from understanding the causal effect of treatment.

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Sounds a bit like omitted-variable bias, right?

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Bigger problem: If selection bias is present, our estimate for τ is biased, preventing us from understanding the causal effect of treatment.

Sounds a bit like omitted-variable bias, right? Our treatment variable is correlated with something that makese the two groups different.

Causality

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Example: Imagine we have two people—Al and Bri—and a single binary treatment, college. We interested in the effect of college on earnings.

$\text{Earn}_{1,\text{Al}} = \$60\text{K}$ $\qquad$ $\text{Earn}_{1,\text{Bri}} = \$140\text{K}$

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but any real-world estimate would have serious selection issues since $\text{Earn}_{0,\text{Al}} \neq \text{Earn}_{0,\text{Bri}}$.

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The selection bias...

If Bri attended college (D_{Bri}=1) and Al did not (D_{Al}=0):

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If Al attended college (D_{Al}=1) and Bri did not (D_{Bri}=0):

$\hat{\tau}$ = $Earn_{1,Al}$ - $Earn_{0,Bri}$ = \$60K - \$110K = -\$50K

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Solution: Eliminate/minimize selection bias.

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- **Option 2: Build a control** group that *matches* the treatment group (life with observational data).

Instrumental variables

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Intro

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Recall: **Selection bias** means our treatment and control groups differ on some unobserved/omitted dimension. (**Endogeneity**)

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- the **exogenous** part of x, which gives us unbiased estimates
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Instrumental variables attempts to separate out

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If we use only the exogenous (*good*) variation in x, then we can avoid selection bias/omitted-variable bias.

Instrumental variables

Introductory example

Example: If we want to estimate the effect of veteran status on earnings,

$$\textbf{Earnings}_i = \beta_0 + \beta_1 \textbf{Veteran}_i + u_i \tag{1}$$

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We would love to calculate $\color{#e64173}{\text{Earnings}_{1i}} - \color{#6A5ACD}{\text{Earnings}_{0i}}$, but we can't.

And OLS will likely be biased for (1) due to selection/omitted-variable bias.

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Instrumental variables

Introductory example

Q: What would exogenous variation in veteran status mean?

A_1: Choices to enlist in the military that are essentially random—or at least uncorrelated with omitted variables and the disturbance.

A_2: No selection bias:

$$Avg(\text{Earnings}_{0i} \mid \text{Veteran}_i = 1) - Avg(\text{Earnings}_{0i} \mid \text{Veteran}_i = 0) = 0$$

Instruments

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Instrumental variables

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Q: What's an instrument?

A: An **instrument** is a variable that is

1. **correlated** with the **explanatory variable** of interest (**relevant**),
2. **uncorrelated** with the **disturbance** (**exogenous**).

Instrumental variables

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So if we want an instrument z_i for endogenous veteran status in

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

1. **Relevant:** $\text{Cov}(\text{Veteran}_i,\ z_i) \neq 0$
2. **Exogenous:** $\text{Cov}(z_i,\ u_i) = 0$

Instruments: Relevance

Relevance: We need the instrument to cause a change in (correlate with) our endogenous explanatory variable.

We can actually test this requirement using regression and a t test.

Instrumental variables

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2. Physical fitness
3. Vietnam War draft

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   being draw led to service

Instrumental variables

Instruments: Exogeneity

Exogeneity: The instrument to be independent of omitted factors that affect our outcome variable—as good as randomly assigned.

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2. Physical fitness — **Not exogenous**
   fitness correlates with many things
3. Vietnam War draft — **Exogenous**
   the lottery was random

Instrumental variables

Instrumental review

Let's recap...

- Our instrument must be **correlated with our endogenous variable**.
- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

Instrumental review

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- Our instrument must be **correlated with our endogenous variable**.
- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

In other words:
The instrument only affects our outcome through the endogenous variable.

Back to our example

For veteran status we considered three potential instruments:

Instrumental variables

Back to our example

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1. Social security number

Not relevant

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Instrumental variables

Back to our example

For veteran status we considered three potential instruments:

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Instrumental variables

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3. Vietnam War draft — **Relevant** **Exogenous**

Back to our example

For veteran status we considered three potential instruments:

1. Social security number — **Not relevant** **Exogenous**
2. Physical fitness — **Probably relevant** **Not exogenous**
3. Vietnam War draft — **Relevant** **Exogenous**

Thus, only the Vietnam War's draft lottery appears to be a ***valid* instrument**.

If we have a *valid* instrument (*e.g.*, the draft lottery), how do we use it?

Estimation

Recall: We want to estimate the effect of veteran status on earnings.

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Instrumental variables

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Let's consider two related effects:

Estimation

Recall: We want to estimate the effect of veteran status on earnings.

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

Let's consider two related effects:

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$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{Draft}_i + v_i$$

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Instrumental variables

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Instrumental variables

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and we know that the draft affected veteran status.

Draft $\longrightarrow$ **Veteran status** $\longrightarrow$ **Earnings**

Using our assumptions on independence and exogeneity:

(Effect of **the draft** on **earnings**) =
(Effect of **the draft** on **veteran status**)×
(Effect of **veteran status** on **earnings**)

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We just wrote out an expression for the effect of **the draft** on **earnings**, *i.e.*,

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Instrumental variables

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Our **instrument** consistently estimates both parts of this fraction!

Instrumental variables

Estimation: Bring it all together

By estimating two regressions involving our **instrument**,

1. The effect of the **instrument** on the **endogenous variable**, *e.g.*,

$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{Draft}_i + v_i$$

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we can estimate our desired effect:

(Effect of **veteran status** on **earnings**) = $\dfrac{\pi_1}{\gamma_1}$

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So with instrumental variables, we estimate β_1 using

$$\hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

where $\hat{\pi}_1$ and $\hat{\gamma}_1$ come from the two equations we just discussed.

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which equals β_1 as long as our instrument is **exogenous** (numerator) and **relevant** (denominator).

Figure 1

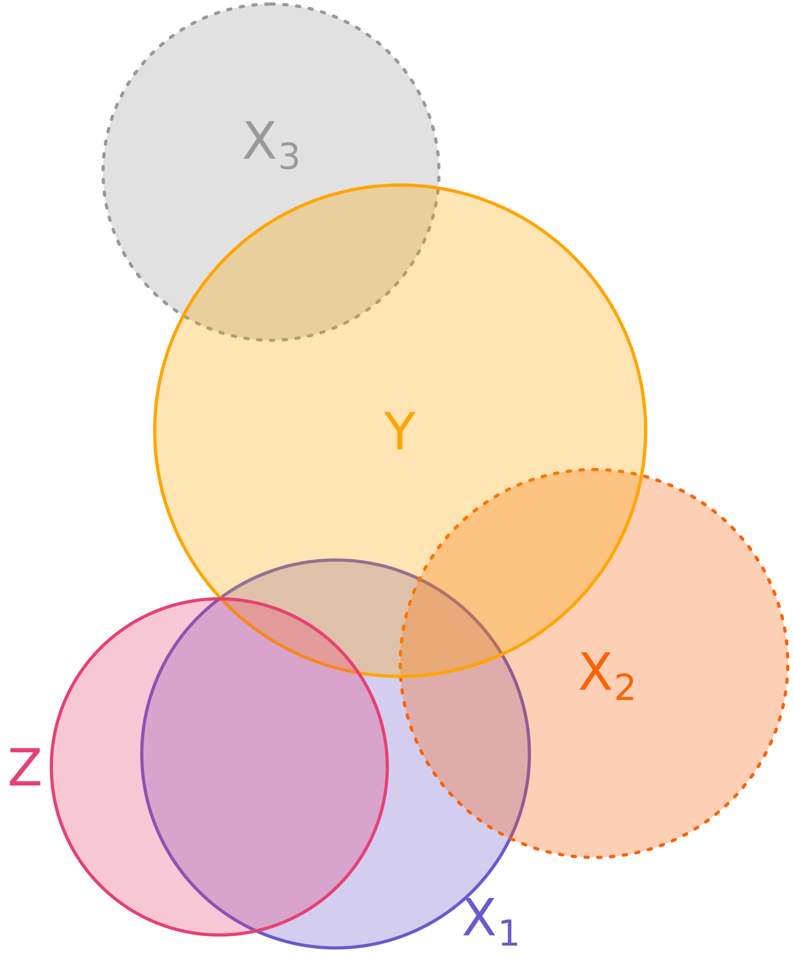


Figure 2

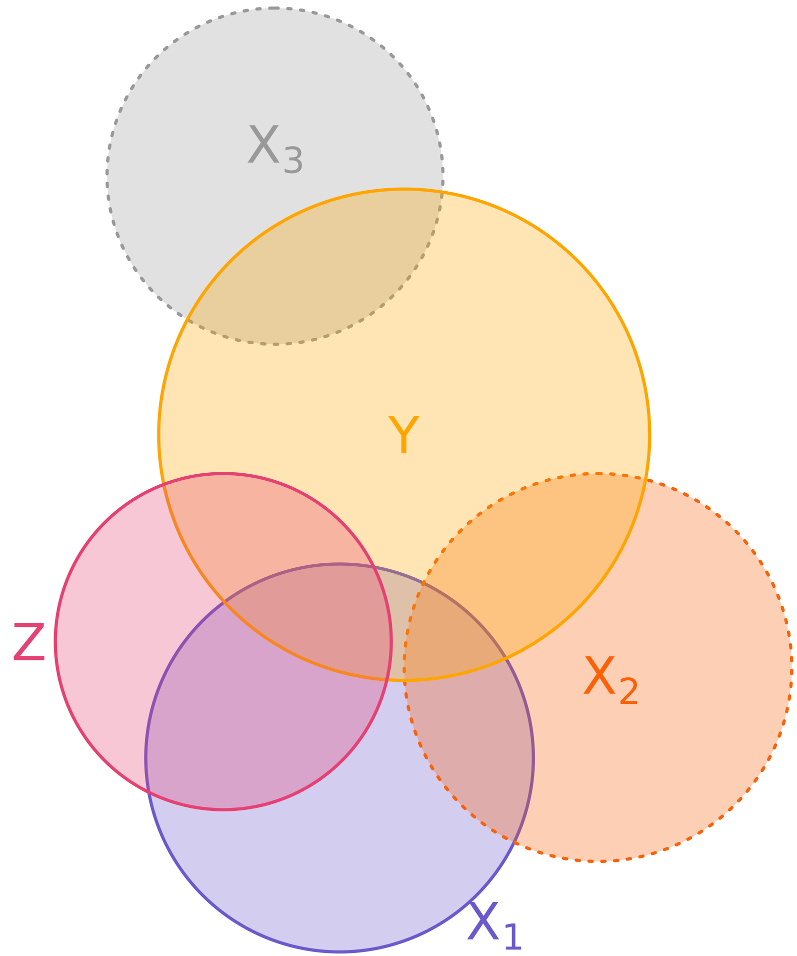


Figure 3

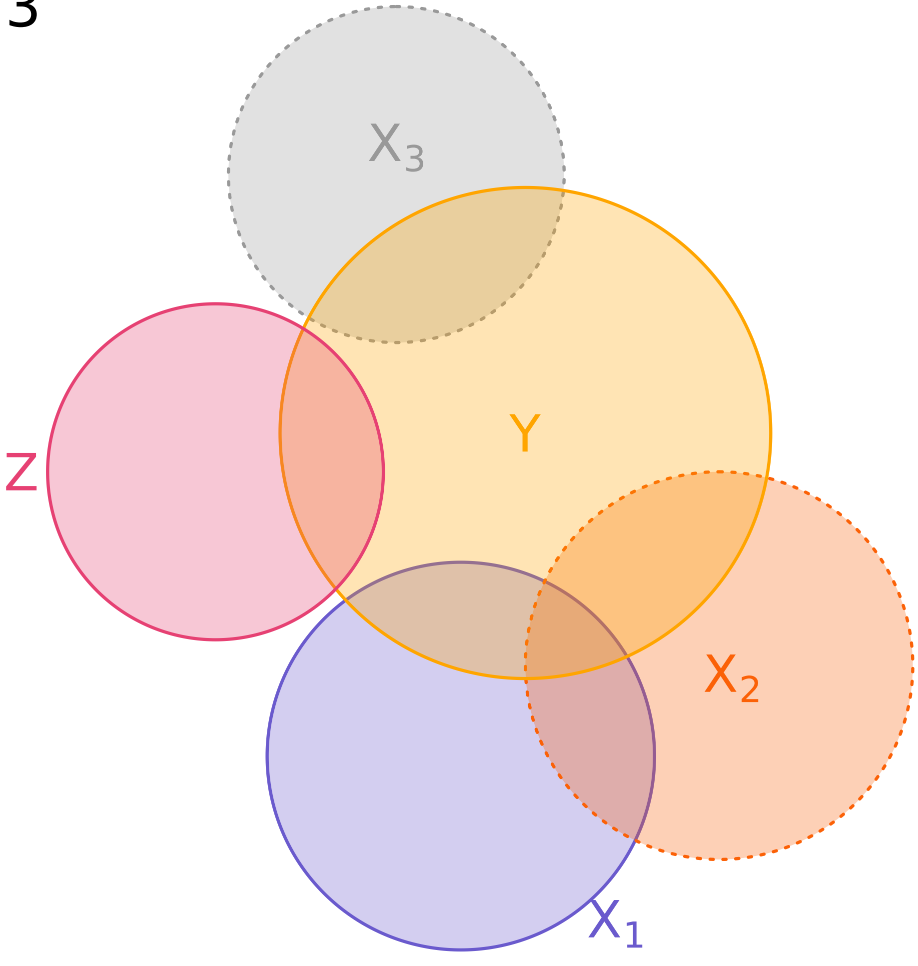
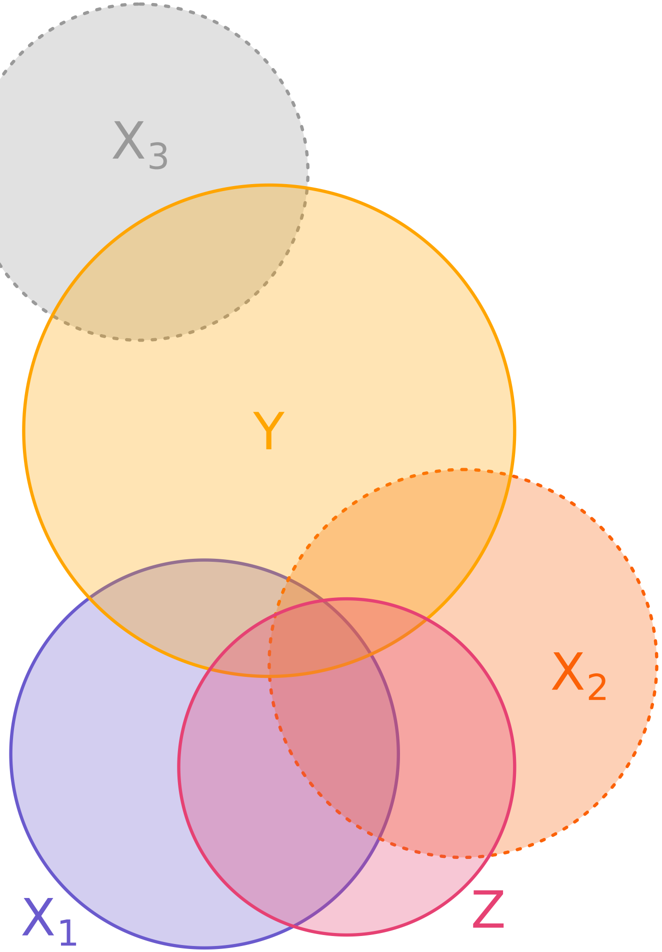


Figure 4



Venn diagram explanation

In these figures (Venn diagrams)

- Each circle illustrates a variable.
- Overlap gives the share of correlatation between two variables.
- Dotted borders denote *omitted* variables.

Take-aways

- Figure 1: **Valid instrument** (relevant; exogenous)
- Figure 2: **Invalid instrument** (relevant; not exogenous)
- Figure 3: **Invalid instrument** (not relevant; not exogenous)
- Figure 4: **Invalid instrument** (relevant; not exogenous)

Let's work an example in **R**.

Example in R

Back to our age-old battle to estimate the returns to education.

```
#> # A tibble: 722 × 4
#>     wage education education_dad education_mom
#>    <int>     <int>         <int>         <int>
#>  1   769        12             8             8
#>  2   808        18            14            14
#>  3   825        14            14            14
#>  4   650        12            12            12
#>  5   562        11            11             6
#>  6   600        10             8             8
#>  7  1154        15             5            14
#>  8  1000        12            11            12
#>  9   930        18            14            13
#> 10   900        15            12            12
#> # ℹ 712 more rows
```

Instrumental variables

Example in R

OLS for the returns to education with will likely (definitely) be biased...

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Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
Education	**58.594**	**6.439**	**9.10**	

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but what if mother's education provides a valid instrument?

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We can check/test the *relevance* of **mother's education** for **education**.

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First-stage results:

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Mother's Education	**0.294**	**0.027**	**10.75**	

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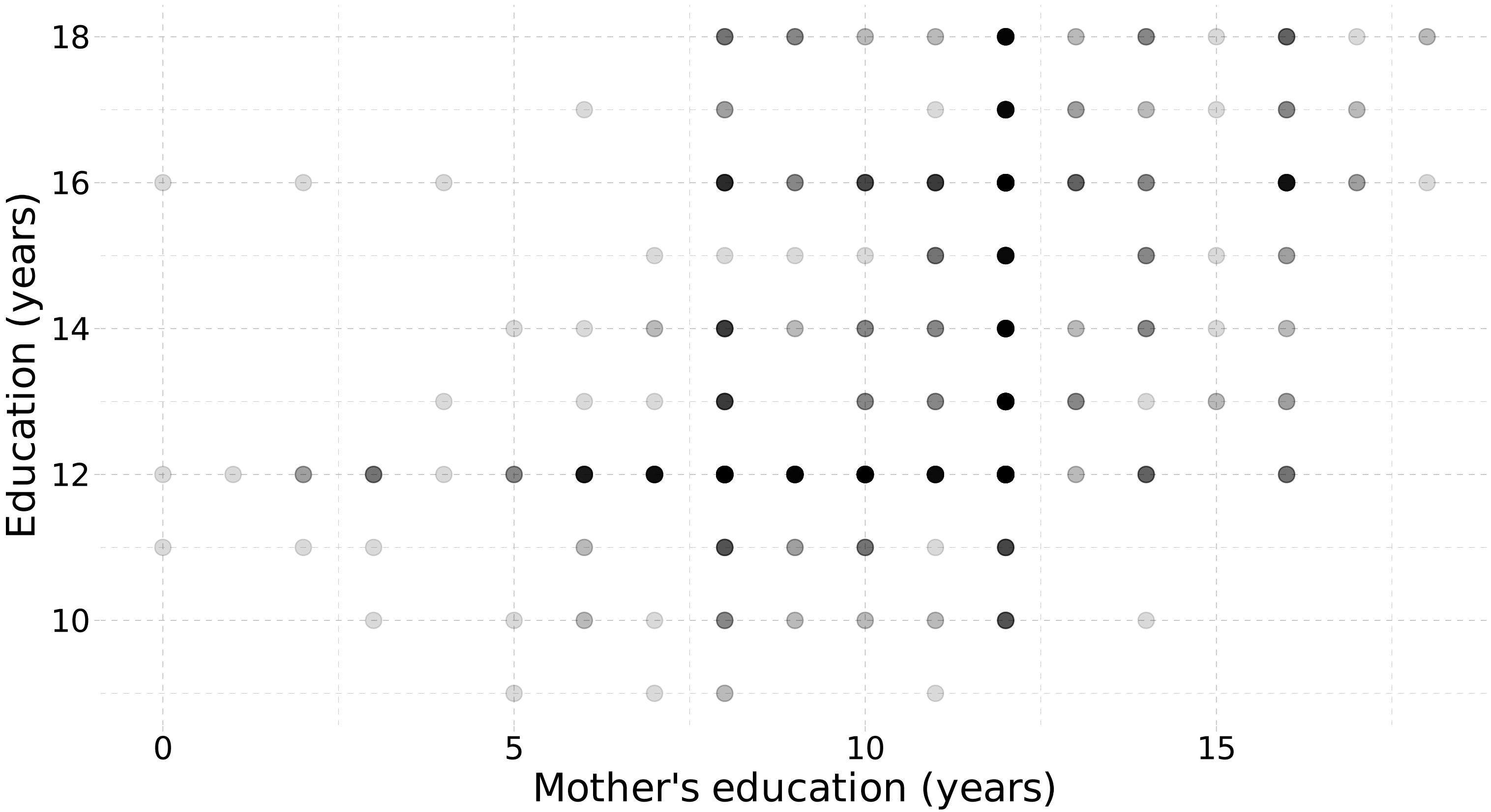
The *p*-value suggests a very strong relationship (very *relevant*).

Instrumental variables

Visualizing the first stage



Instrumental variables

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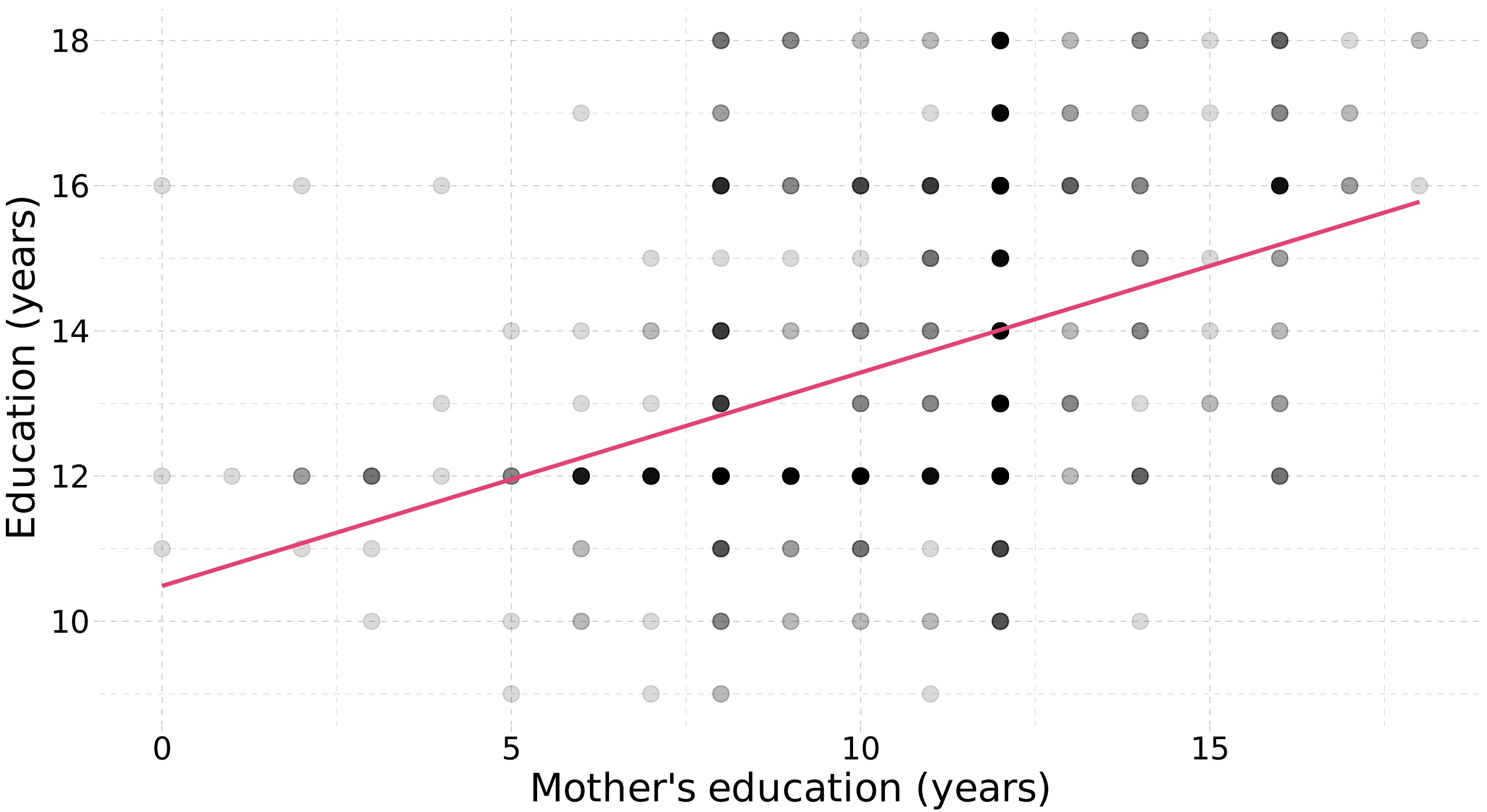

Exogeneity

Q: What does **exogeneity** mean in this case?

Instrumental variables

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A: We need two things

1. Mother's education (our instrument) must only affect earnings through education (our endogenous explanatory variable).
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Q: Does *mother's education* seem likely to satisfy exogeneity?

Instrumental variables

Example in R

Now, let's estimate the ***reduced form***:

The effect of the instrument on our outcome variable.

$$\text{Wage}_i = \pi_0 + \pi_1 (\text{Mother's Education})_i + w_i$$

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Q_2: If our instrument is *valid*, can we interpret these estimates as **causal**?

Instrumental variables

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2. In the **first-stage equation**, we estimated $\hat{\gamma}_1 \approx 0.294$.

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$$\implies \hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{31.81}{0.294} \approx 108.2$$

Instrumental variables

Example in R

Alternative: Use the function `iv_robust()` from the `estimatr` package.

This new function `iv_robust` works very similar to our good friend `lm`:

```
iv_robust(y ~ x | z, data = dataset)
```

- `formula` Specify the regression followed by `|` and your instrument (`z`).
- `data` You still need a dataset.

Note: As you might guess by its name, `iv_robust` calculates heteroskedasticity-robust standard errors by default.

Example in R

In practice...

```r
# Estimate our IV regression
iv_est ← iv_robust(wage ~ education | education_mom, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
Education	**108.214**	**16.810**	**6.44**	

Instrumental variables

More

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Instrumental variables

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1. Estimate the reduced form (regress outcome var. on instrument).
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A: Too bad.

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A: ~~Too bad.~~ Extend IV to **two-stage least squares (2SLS)**.

Two-stage least squares

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Intro

The intuition and insights of IV carry over into two-stage least squares.

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Plus: The *first stage* that we've been discussing is actually the *first* of the *two stages* in two-stage least squares.

$$\begin{aligned} \text{Endogenous model} \qquad \text{Outcome}_i &= \beta_0 + \beta_1 \left(\text{Endog. var.}\right)_i + u_i \\ \text{First stage} \qquad \left(\text{Endog. var.}\right)_i &= \pi_0 + \pi_1 \text{Instrument}_i + v_i \\ \text{Second stage} \qquad \text{Outcome}_i &= \delta_0 + \delta_1 \widehat{\left(\text{Endog. var.}\right)}_i + \varepsilon_i \\ \text{Reduced form} \qquad \text{Outcome}_i &= \pi_0 + \pi_1 \text{Instrument}_i + w_i \end{aligned}$$

where $\widehat{\left(\text{Endog. var.}\right)}_i$ denotes the predicted values (*fitted values*) from the first-stage regression.

Two-stage least squares

Intro

Two-stage least squares is very flexible—we include other controls, additional endogenous variables, *and* have multiple instruments.

But don't get too distracted by this fancy flexiblity.

We still need **valid** instruments.

Two-stage least squares

In R

Back to our *returns to education* example.

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + u_i$$

Imagine that mother's *and* father's education are both valid instruments.

Then our **first-stage regression** is

$$\text{Education}_i = \gamma_0 + \gamma_1 \left(\text{Mother's education}\right)_i + \gamma_2 \left(\text{Father's education}\right)_i + v_i$$

which we can estimate via OLS.

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Q: Why?

Two-stage least squares

In R

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's education})_i + \gamma_2 (\text{Father's education})_i + v_i$$

```
stage1 <- lm(education ~ education_mom + education_dad, wage_df)
```

First-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	9.845	0.305	32.31	
Mother's Education	**0.149**	**0.032**	**4.62**	
Father's Education	**0.216**	**0.028**	**7.84**	

Two-stage least squares

In R

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Formally, we should jointly test them (*e.g.*, F test).

Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

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# Add fitted values from first stage
wage_df$education_hat <- stage1$fitted.values
```

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```r
# Add fitted values from first stage
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```

Now we use OLS (again) to estimate the **second-stage regression**

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

Two-stage least squares

In R

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

```
stage2 <- lm(wage ~ education_hat, wage_df)
```

Second-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
Fitted Education	**104.789**	**14.462**	**7.25**	

Ordinary least squares

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
Education	**58.594**	**6.439**	**9.10**	

Instrumental variables

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
Education	**108.214**	**16.810**	**6.44**	

Two-stage least squares w/ two instruments

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
Education	**104.789**	**14.462**	**7.25**	

Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

Two-stage least squares

In R

As you probably guessed, **R** will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + … | z1 + z2 + …, data)
```

Two-stage least squares

In R

As you probably guessed, **R** will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + … | z1 + z2 + …, data)
```

In our case, we have

- one explanatory variable (`x`) (education)
- two instruments (`z`) (parents' educations)

```
iv_robust(wage ~ education | education_mom + education_dad, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	199.946	-2.27	0.0233
Education, fitted	**104.789**	**14.852**	**7.06**	

Two-stage least squares

There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

Two-stage least squares

There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

Common applications

- **General causal inference** for observational data (as we've seen).
- **Experiments:** Randomize a treatment that affects an endog. variable.
- **Measurement error:** Regress noisy x_1 on noisy x_2 to capture signal.
- **Simultaneous relationships** (*e.g.*, p and q from supply and demand).

However, in any 2SLS/IV setting, you need to mind the requirements for **valid instruments**—exogeneity and relevance.

Table of contents

Admin

Instrumental variables