

# Non-Stationary Time Series

EC 421, Set 9

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# Prologue

# Schedule

## Last Time

Autocorrelation

## Today

Brief introduction to nonstationarity

## Upcoming

Assignment soon.

# Nonstationarity

# Nonstationarity

## Intro

Let's go back to our assumption of **weak dependence/persistence**

1. **Weakly persistent outcomes**—essentially,  $x_{t+k}$  in the distant period  $t + k$  weakly correlates with  $x_t$  (when  $k$  is "big").

We're essentially saying we need the time series  $x$  to behave.

We'll define this *good behavior* as **stationarity**.

# Nonstationarity

## Stationarity

Requirements for **stationarity** (a *stationary* time-series process):

1. The **mean** of the distribution is independent of time, *i.e.*,

$$\mathbf{E}[x_t] = \mathbf{E}[x_{t-k}] \text{ for all } k$$

2. The **variance** of the distribution is independent of time, *i.e.*,

$$\text{Var}(x_t) = \text{Var}(x_{t-k}) \text{ for all } k$$

3. The **covariance** between  $x_t$  and  $x_{t-k}$  depends only on  $k$ —**not on  $t$** , *i.e.*,

$$\text{Cov}(x_t, x_{t-k}) = \text{Cov}(x_s, x_{s-k}) \text{ for all } t \text{ and } s$$

# Nonstationarity

## Random walks

**Random walks** are a famous example of a nonstationary process:

$$x_t = x_{t-1} + \varepsilon_t$$

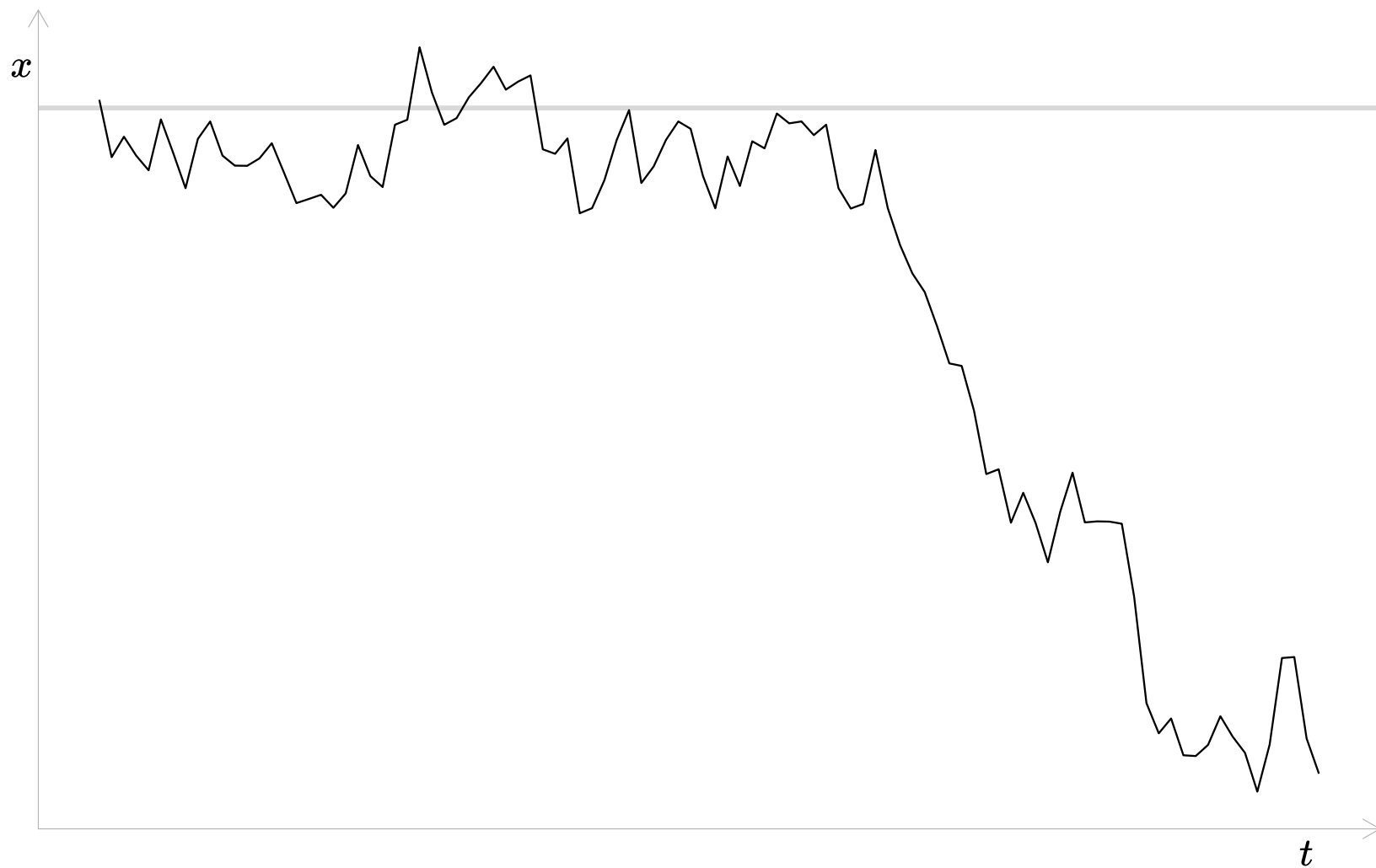
Why?  $\text{Var}(x_t) = t\sigma_\varepsilon^2$ , which **violates stationary variance**.

$$\begin{aligned}\text{Var}(x_t) &= \text{Var}(x_{t-1} + \varepsilon_t) \\ &= \text{Var}(x_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\ &= \text{Var}(x_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\ &\dots \\ &= \text{Var}(x_0 + \varepsilon_1 + \dots + \varepsilon_{t_2} + \varepsilon_{t-1} + \varepsilon_t) \\ &= \sigma_\varepsilon^2 + \dots + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \\ &= t\sigma_\varepsilon^2\end{aligned}$$

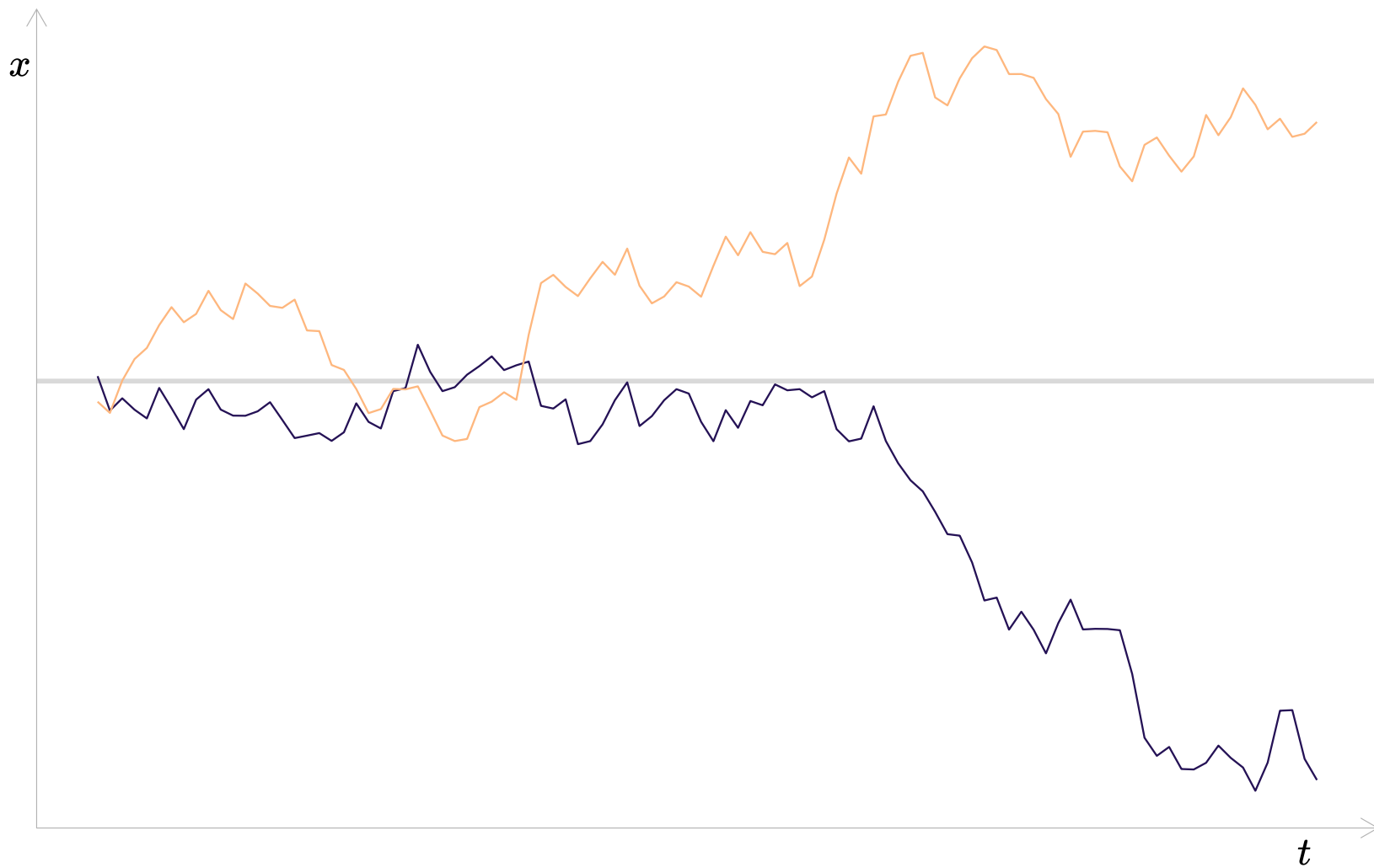
**Q:** What's the big deal with this violation?



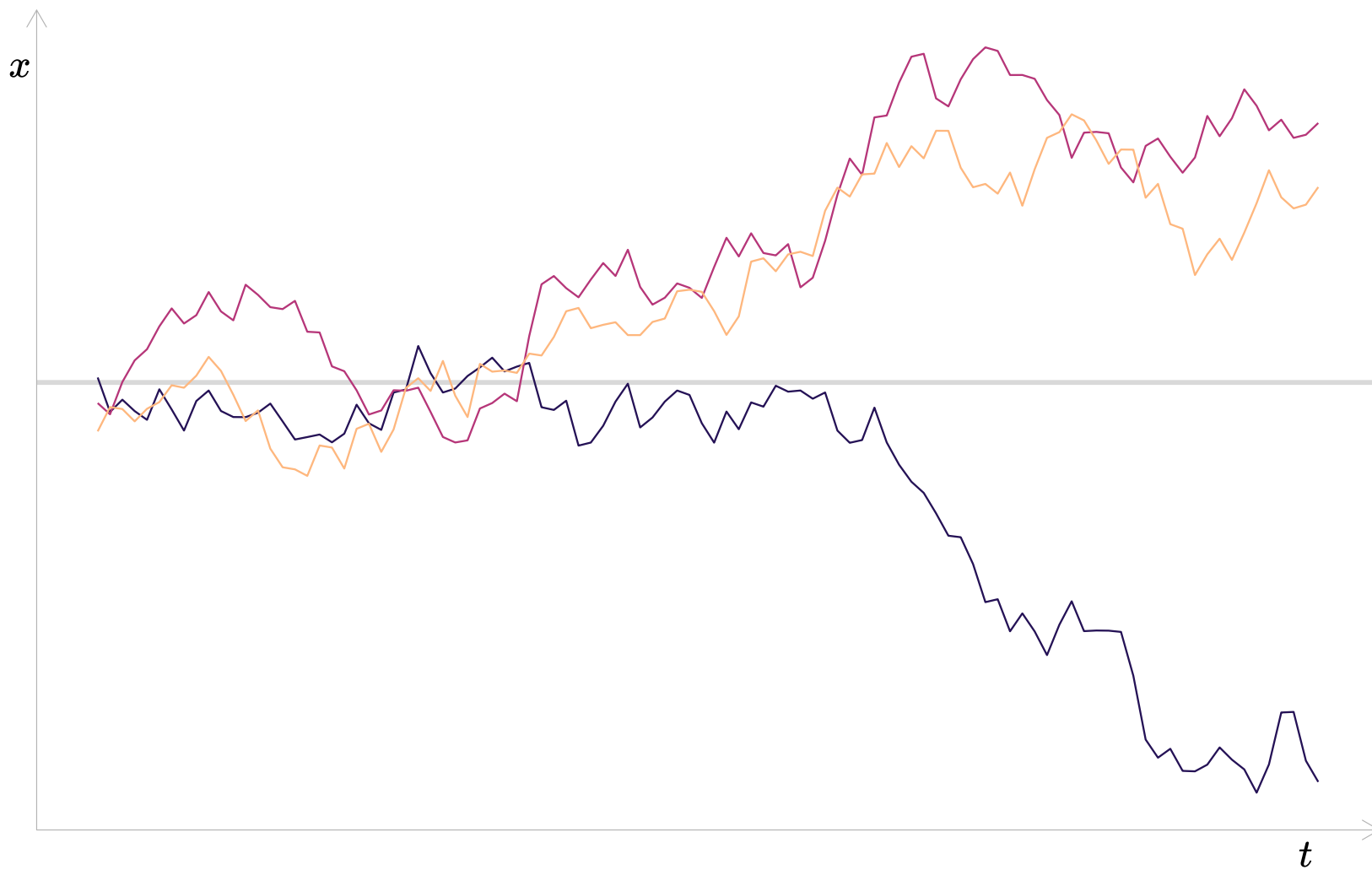
## One 100-period random walk



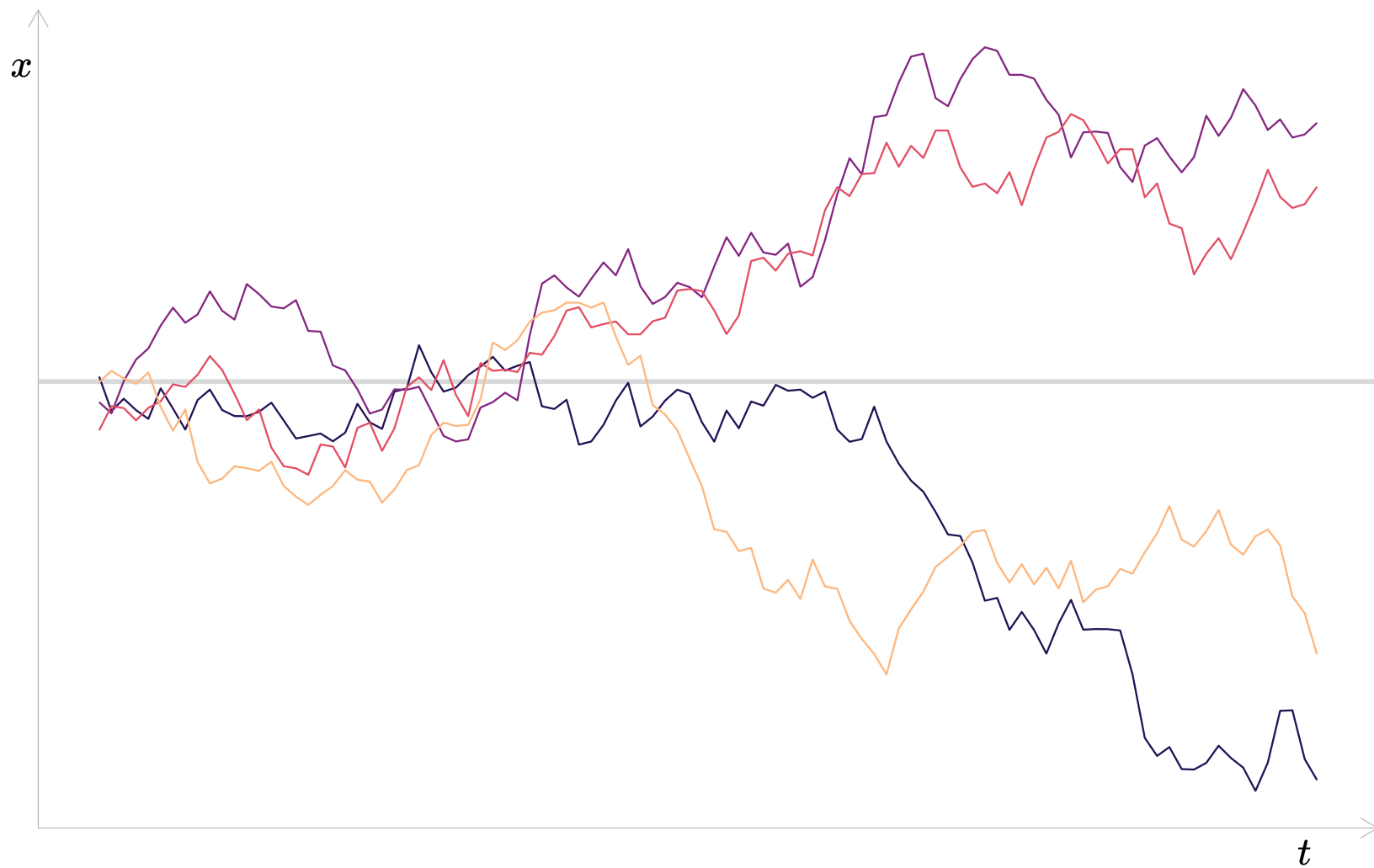
## Two 100-period random walks



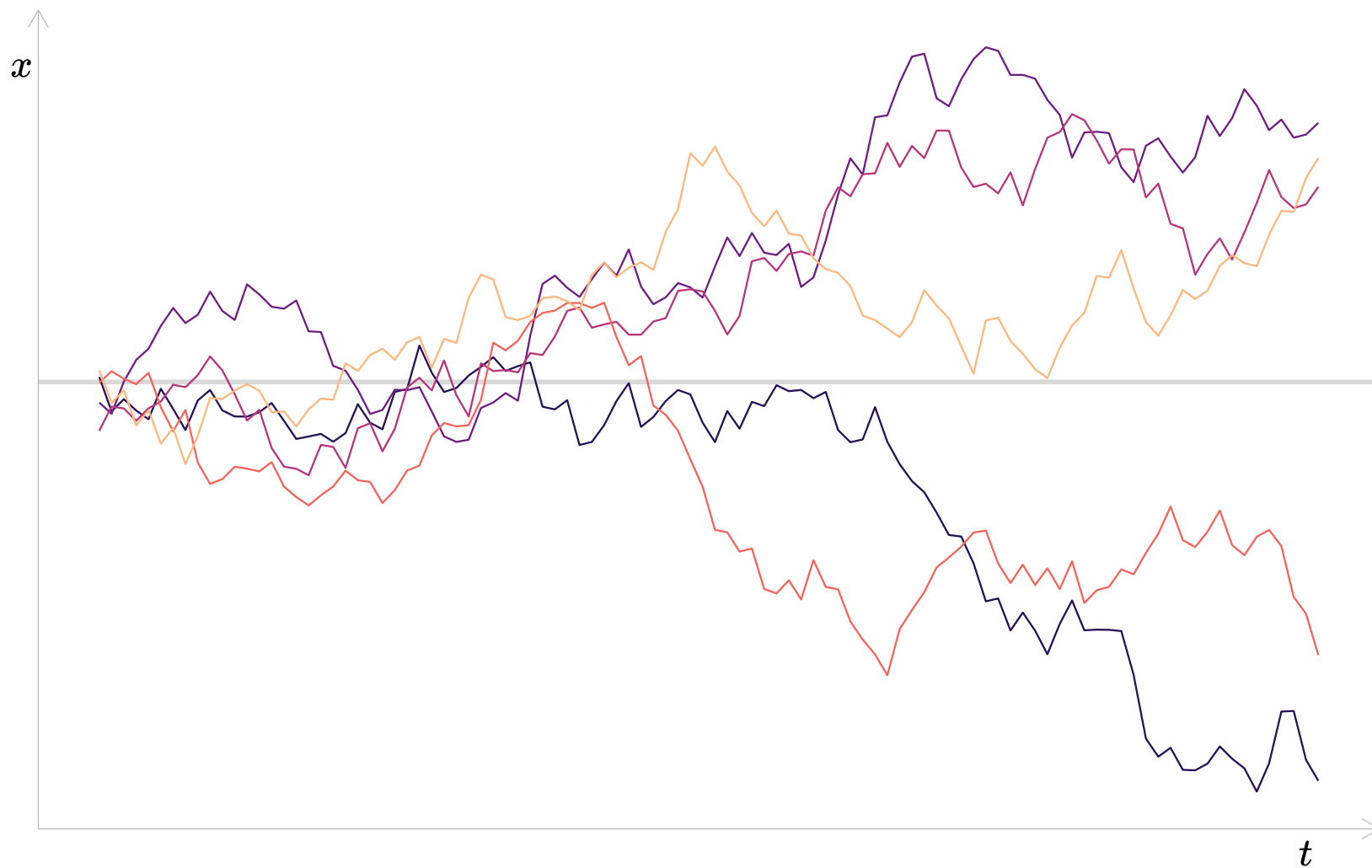
## Three 100-period random walks



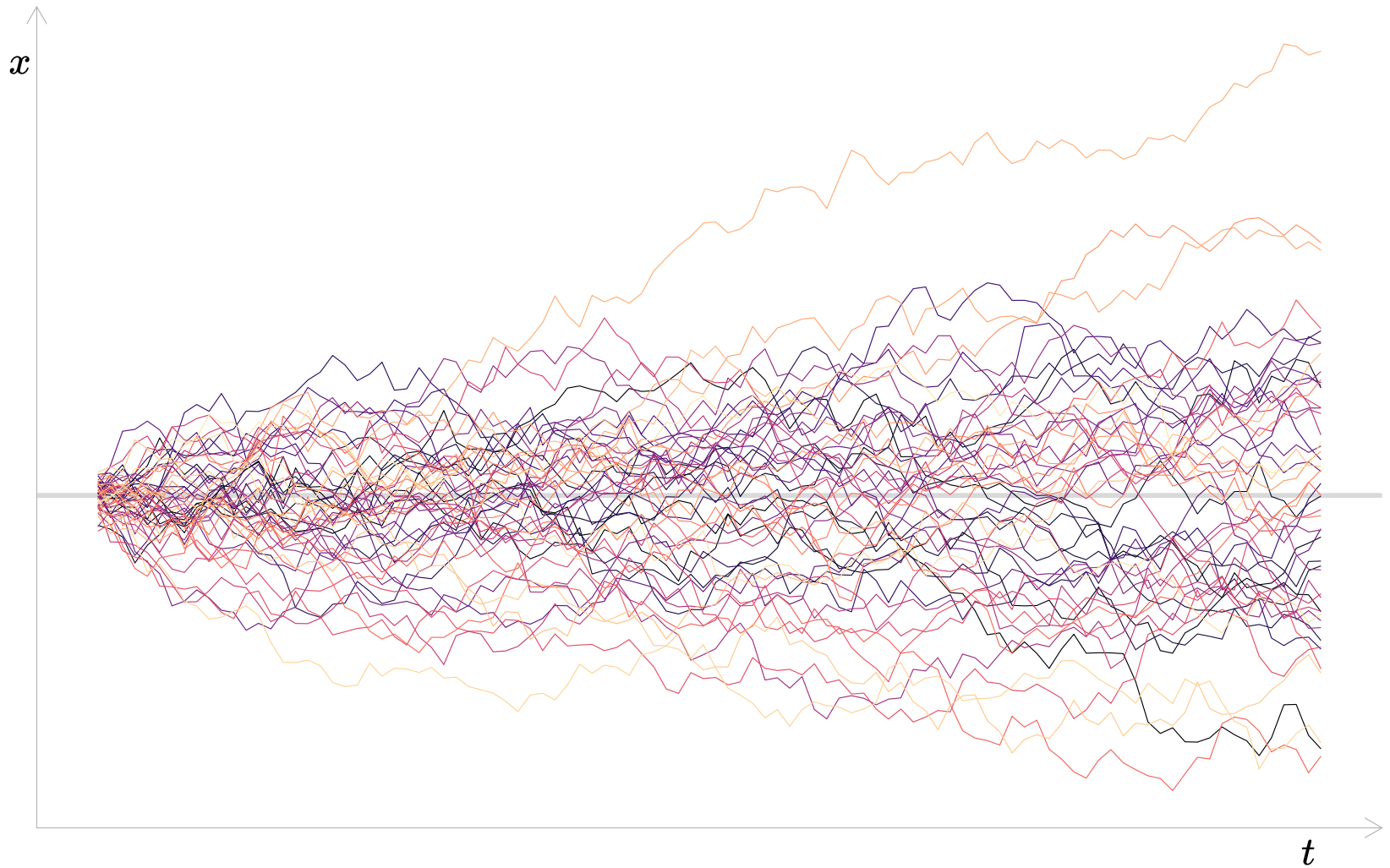
## Four 100-period random walks



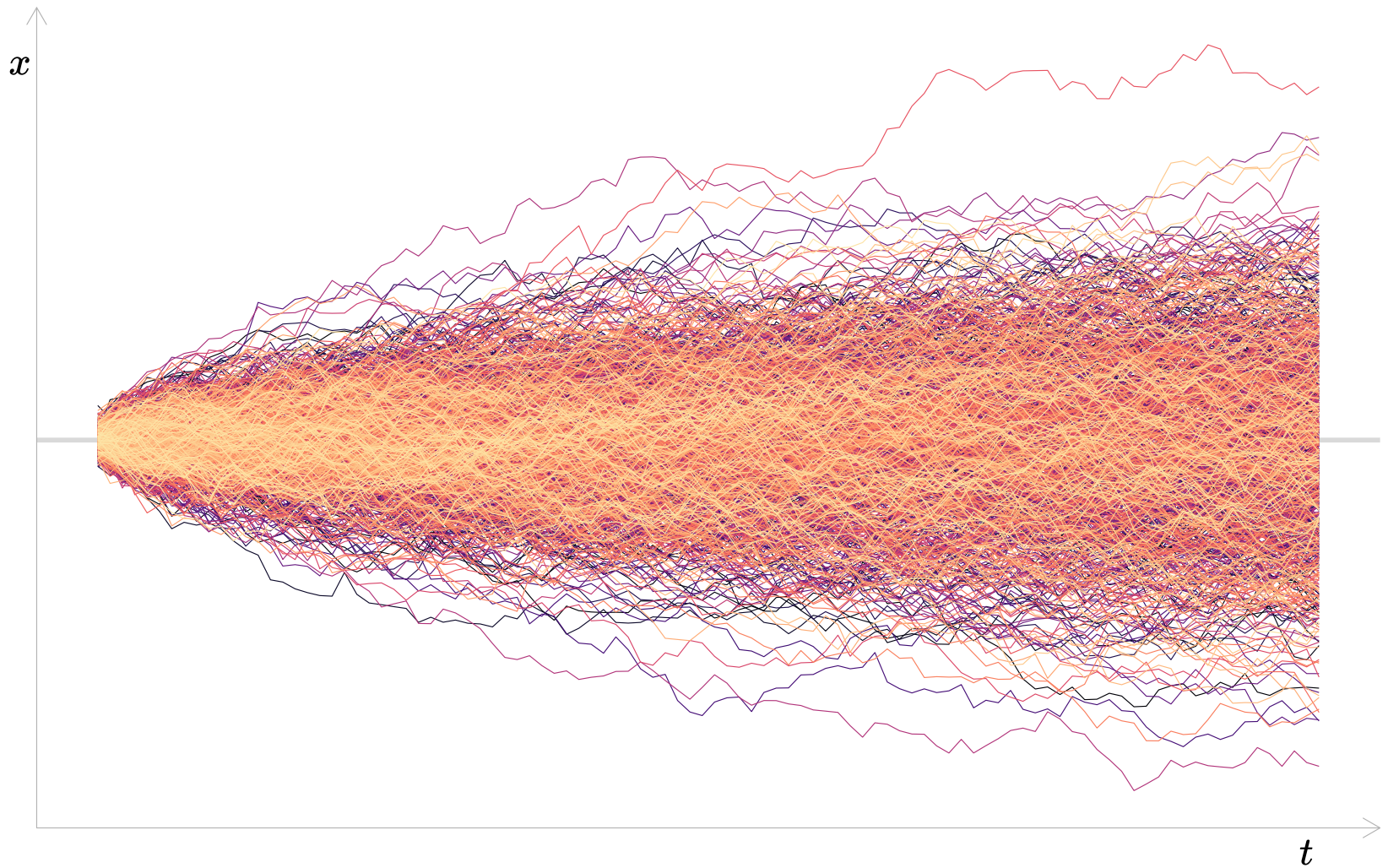
## Five 100-period random walks



## Fifty 100-period random walks



## 1,000 100-period random walks



# Nonstationarity

## Problem

One problem is that nonstationary processes can lead to **spurious** results.

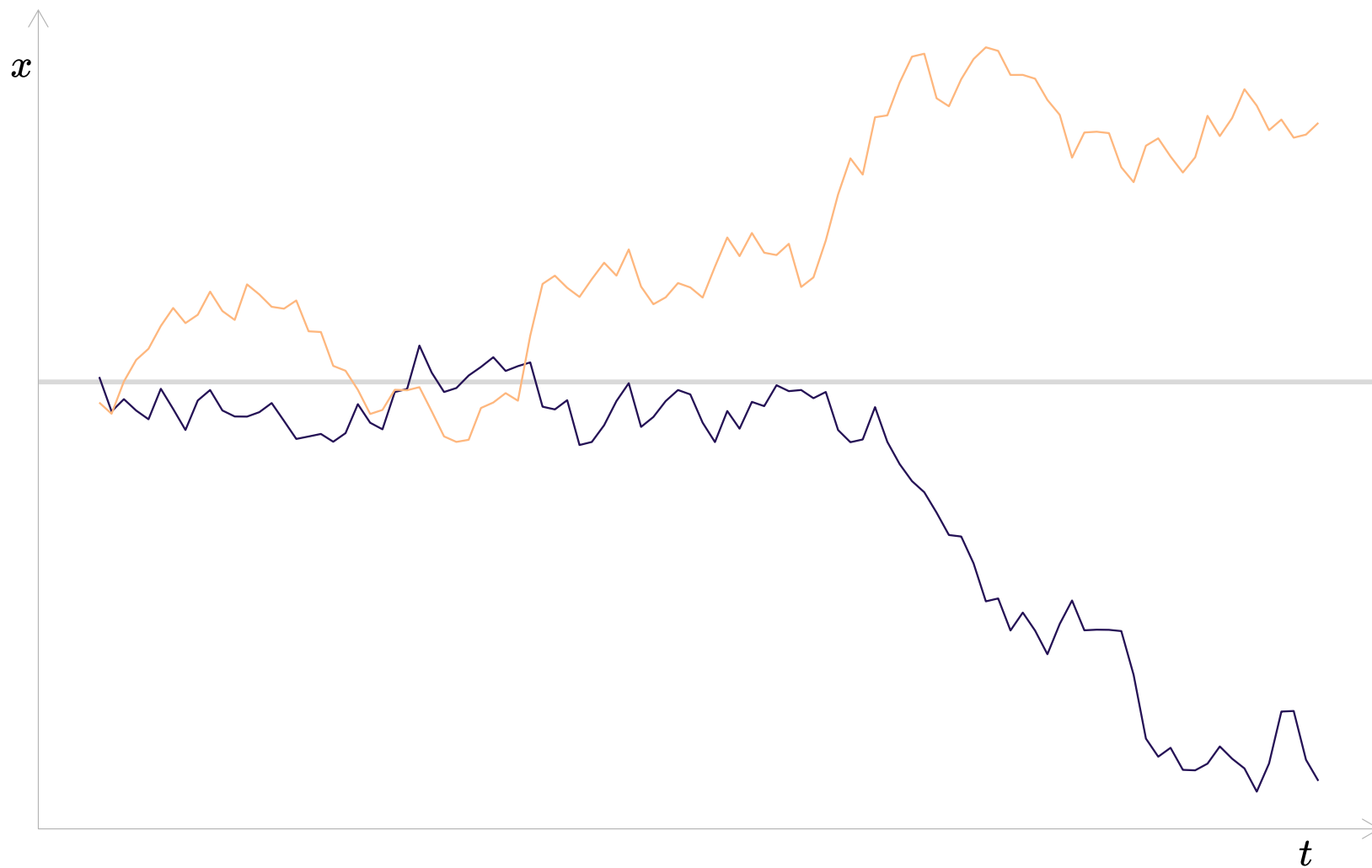
### **Defintion: Spurious**

- not being what it purports to be; false or fake
- apparently but not actually valid

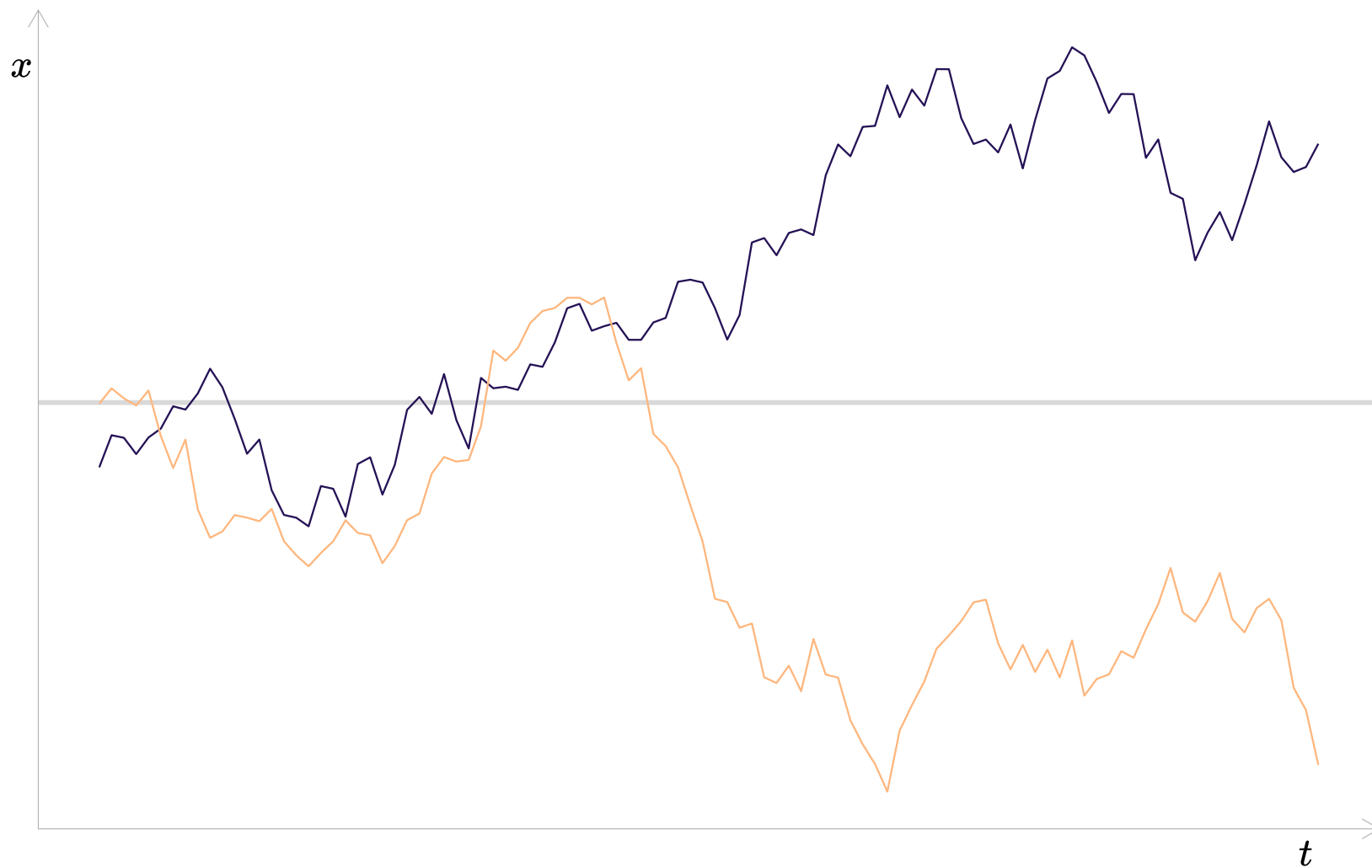
Back in 1974, Granger and Newbold showed that when they **generated random walks** and **regressed the random walks on each other**, **77/100 regressions were statistically significant** at the 5% level (should have been approximately 5/100).



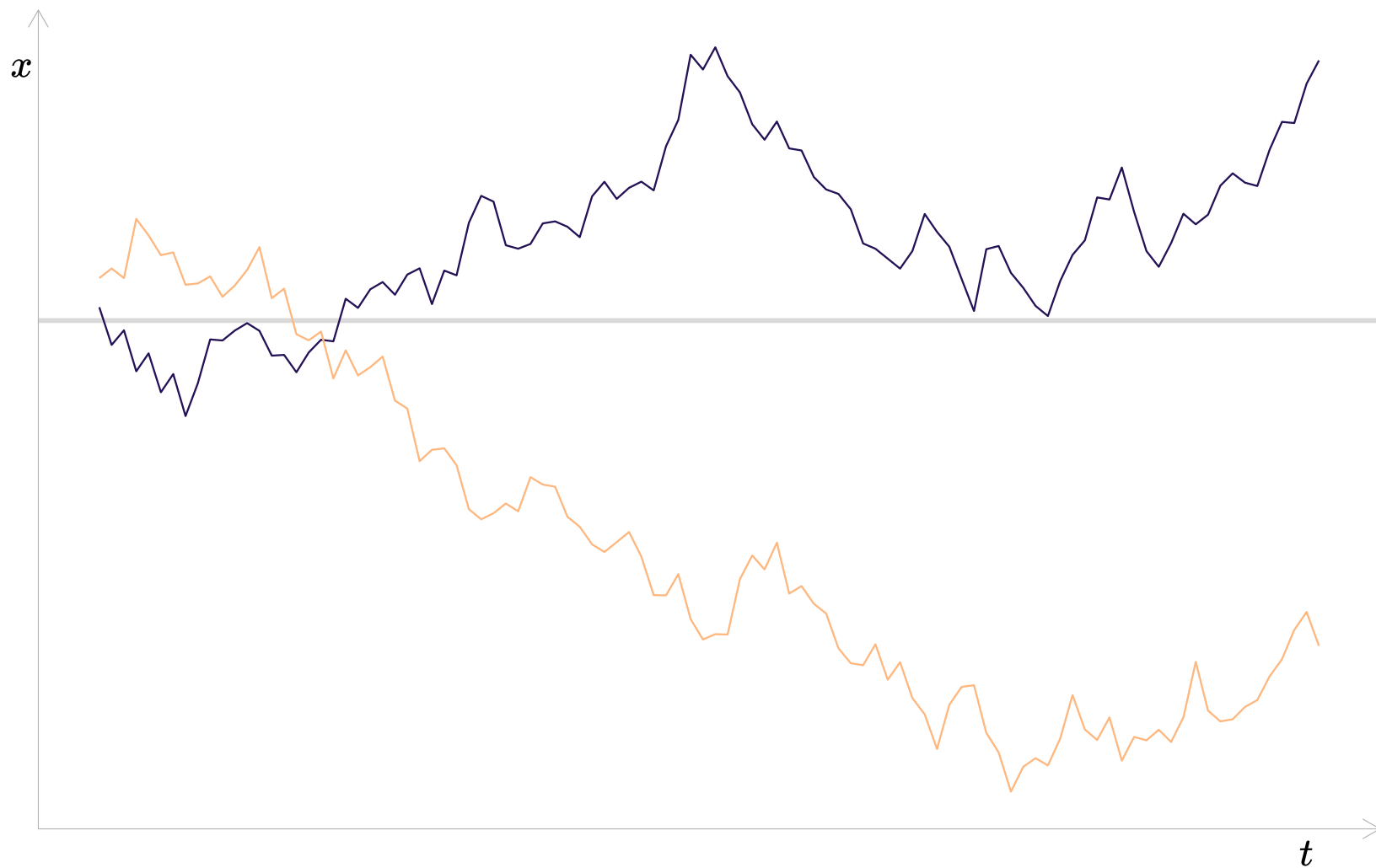
## Granger and Newbold simulation example: $t$ statistic $\approx -10.58$



## Granger and Newbold simulation example: $t$ statistic $\approx -8.92$



## Granger and Newbold simulation example: $t$ statistic $\approx -7.23$



# Nonstationarity

## Problem

In our data, 74.6 percent of (independently generated) pairs reject the null hypothesis at the 5% level.

**The point?** If our disturbance is nonstationary, we cannot trust plain OLS.

Random walks are only one example of nonstationary processes...

**Random walk:**  $u_t = u_{t-1} + \varepsilon_t$

**Random walk with drift:**  $u_t = \alpha_0 + u_{t-1} + \varepsilon_t$

**Deterministic trend:**  $u_t = \alpha_0 + \beta_1 t + \varepsilon_t$

# Nonstationarity

## A potential solution

Some processes are **difference stationary**, which means we can get back to our stationarity (good behavior) requirement by taking the difference between  $u_t$  and  $u_{t-1}$ .

**Nonstationary:**  $u_t = u_{t-1} + \varepsilon_t$  (a random walk)

**Stationary:**  $u_t - u_{t-1} = u_{t-1} + \varepsilon_t - u_{t-1} = \varepsilon_t$

So if we have good reason to believe that our disturbances follow a random walk, we can use OLS on the differences, *i.e.*,

$$\begin{aligned}y_t &= \beta_0 + \beta_1 x_t + u_t \\y_{t-1} &= \beta_0 + \beta_1 x_{t-1} + u_{t-1} \\y_t - y_{t-1} &= \beta_1 (x_t - x_{t-1}) + (u_t - u_{t-1}) \\\Delta y_t &= \beta_1 \Delta x_t + \Delta u_t\end{aligned}$$

# Nonstationarity

## Testing

Dickey-Fuller and augmented Dickey-Fuller tests are popular ways to test of random walks and other forms of nonstationarity.

**Dickey-Fuller tests** compare

$H_0: y_t = \beta_0 + \beta_1 y_{t-1} + u_t$  with  $|\beta_1| < 1$  (**stationarity**)

$H_a: y_t = y_{t-1} + \varepsilon_t$  (**random walk**)

using a  $t$  test that  $|\beta_1| < 1$ .<sup>†</sup>

<sup>†</sup> People often just test  $\beta_1 < 1$ .

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