Instrumental Variables

EC 421, Set 11

Edward Rubin

08 March 2019

Prologue

Schedule

Last Time

Causality

Today

- Econ. Masters program
- Review: Causality
- New: Instrumental variables

Upcoming

Assignment soon.

Master's Program

Applied Economics

You could be a master of (applied) economics...

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- Awesome opportunity to focus on applying economic methods to real-world questions/scenarios.
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Causality

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or we observe i in the control group, *i.e.*,

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but never both at the same time.

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Idea: Estimate the **average treatment effect** as the difference between the average outcomes in the treatment group and the control group, *i.e.*,

$$Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0)$$

where $D_i = 1$ if i received treatment, and $D_i = 0$ if i is in the control group.

Review

Result: We showed that even when the treatment effect is constant (meaning $\tau_i = \tau$ for all i),

$$\begin{aligned} & Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0) \\ & = \tau + \underbrace{Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)}_{\text{Selection bias}} \end{aligned}$$

which says that the difference in the groups' means will give us a **biased estimate** for the causal effect of treatment **if we have selection bias.**

Causality

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A: (Formal) The average *untreated* outcome for a member of our **treatment group** (which we cannot observe) differs from the average *untreated* outcome for a member of our **control group**, *i.e.*,

$$Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)$$

Causality

Review

Practical problem: Selection bias is also difficult to observe

$$\underbrace{Avg(y_{0,i} \mid D_i = 1)}_{\text{Unobservable}} - Avg(y_{0,i} \mid D_i = 0)$$

(back to the *fundamental problem of causal inference*)

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Bigger problem: If selection bias is present, our estimate for τ is biased, preventing us from understanding the causal effect of treatment.

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Sounds a bit like omitted-variable bias, right?

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Bigger problem: If selection bias is present, our estimate for τ is biased, preventing us from understanding the causal effect of treatment.

Sounds a bit like omitted-variable bias, right? Our treatment variable is correlated with something that makese the two groups different.

Causality

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Example: Imagine we have two people—Al and Bri—and a single binary treatment, college. We interested in the effect of college on earnings.

$\text{Earn}_{1,\text{Al}} = \$60\text{K}$ $\qquad$ $\text{Earn}_{1,\text{Bri}} = \$140\text{K}$

$\text{Earn}_{0,\text{Al}} = \$30\text{K}$ $\qquad$ $\text{Earn}_{0,\text{Bri}} = \$110\text{K}$

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but any real-world estimate would have serious selection issues since $\text{Earn}_{0,\text{Al}} \neq \text{Earn}_{0,\text{Bri}}$.

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If Al attended college (D_{Al}=1) and Bri did not (D_{Bri}=0):

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- **Option 1:** **Distribute treatment** in a way such that the treatment and control groups are essentially identical (experiments).
- **Option 2:** **Build a control** group that *matches* the treatment group (life with observational data).

Instrumental variables

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Intro

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Recall: **Selection bias** means our treatment and control groups differ on some unobserved/omitted dimension. (**Endogeneity**)

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Recall: **Selection bias** means our treatment and control groups differ on some unobserved/omitted dimension. (**Endogeneity**)

Instrumental variables attempts to separate out

- the **exogenous** part of x, which gives us unbiased estimates
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If we use only the exogenous (*good*) variation in x, then we can avoid selection bias/omitted-variable bias.

Introductory example

Example: If we want to estimate the effect of veteran status on earnings,

$$\mathbf{Earnings}_i = \beta_0 + \beta_1 \mathbf{Veteran}_i + u_i \tag{1}$$

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And OLS will likely be biased for (1) due to selection/omitted-variable bias.

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&= \beta_0 + \beta_1 \left(\textbf{Veteran}_i^{\text{Exog.}} + \textbf{Veteran}_i^{\text{Endog.}} \right) + u_i \\
&= \beta_0 + \beta_1 \textbf{Veteran}_i^{\text{Exog.}} + \underbrace{\beta_1 \textbf{Veteran}_i^{\text{Endog.}} + u_i}_{w_i} \\
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Q: What would exogenous variation in veteran status mean?

A_1: Choices to enlist in the military that are essentially random—or at least uncorrelated with omitted variables and the disturbance.

A_2: **No selection bias:**

$$Avg(\text{Earnings}_{0i} \mid \text{Veteran}_i = 1) - Avg(\text{Earnings}_{0i} \mid \text{Veteran}_i = 0) = 0$$

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A: An **instrument** is a variable that is

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So if we want an instrument z_i for endogenous veteran status in

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

1. **Relevant:** $\text{Cov}(\text{Veteran}_i,\, z_i) \neq 0$
2. **Exogenous:** $\text{Cov}(z_i,\, u_i) = 0$

Instrumental variables

Instruments: Relevance

Relevance: We need the instrument to cause a change in (correlate with) our endogenous explanatory variable.

We can actually test this requirement using regression and a t test.

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3. Vietnam War draft

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3. Vietnam War draft — **Relevant**
   being draw led to service

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Exogeneity: The instrument to be independent of omitted factors that affect our outcome variable—as good as randomly assigned.

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   fitness correlates with many things

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2. Physical fitness — **Not exogenous**
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3. Vietnam War draft — **Exogenous**
   the lottery was random

Instrumental review

Let's recap...

- Our instrument must be **correlated with our endogenous variable**.
- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

Instrumental review

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- Our instrument must be **correlated with our endogenous variable**.
- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

In other words:

The instrument only affects our outcome through the endogenous variable.

Back to our example

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Exogenous

Back to our example

For veteran status we considered three potential instruments:

1. Social security number — **Not relevant** / **Exogenous**

2. Physical fitness — **Probably relevant** / **Not exogenous**

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3. Vietnam War draft — **Relevant** **Exogenous**

Back to our example

For veteran status we considered three potential instruments:

1. Social security number — **Not relevant**, **Exogenous**
2. Physical fitness — **Probably relevant**, **Not exogenous**
3. Vietnam War draft — **Relevant**, **Exogenous**

Thus, only the Vietnam War's draft lottery appears to be a ***valid* instrument**.

If we have a *valid* instrument (*e.g.*, the draft lottery), how do we use it?

Instrumental variables

Estimation

Recall: We want to estimate the effect of veteran status on earnings.

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Instrumental variables

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Draft $\longrightarrow$ **Veteran status** $\longrightarrow$ **Earnings**

Using our assumptions on independence and exogeneity:

(Effect of **the draft** on **earnings**) =

(Effect of **the draft** on **veteran status**)×

(Effect of **veteran status** on **earnings**)

Instrumental variables

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We just wrote out an expression for the effect of **the draft** on **earnings**, *i.e.*,

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(Effect of **veteran status** on **earnings**) =

$$\frac{\text{(Effect of the draft on earnings)}}{\text{(Effect of the draft on veteran status)}}$$

Instrumental variables

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$$\text{(Effect of \textbf{veteran status} on \textbf{earnings})} = \frac{\text{(Effect of \textbf{the draft} on \textbf{earnings})}}{\text{(Effect of \textbf{the draft} on \textbf{veteran status})}}$$

Our **instrument** consistently estimates both parts of this fraction!

Estimation: Bring it all together

By estimating two regressions involving our **instrument**,

1. The effect of the **instrument** on the **endogenous variable**, *e.g.*,

$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{Draft}_i + v_i$$

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we can estimate our desired effect:

(Effect of **veteran status** on **earnings**) = $\dfrac{\pi_1}{\gamma_1}$

Estimation: Bring it all together

So with instrumental variables, we estimate β_1 using

$$\hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

where $\hat{\pi}_1$ and $\hat{\gamma}_1$ come from the two equations we just discussed.

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which equals β_1 as long as our instrument is **exogenous** (numerator) and **relevant** (denominator).

Figure 1

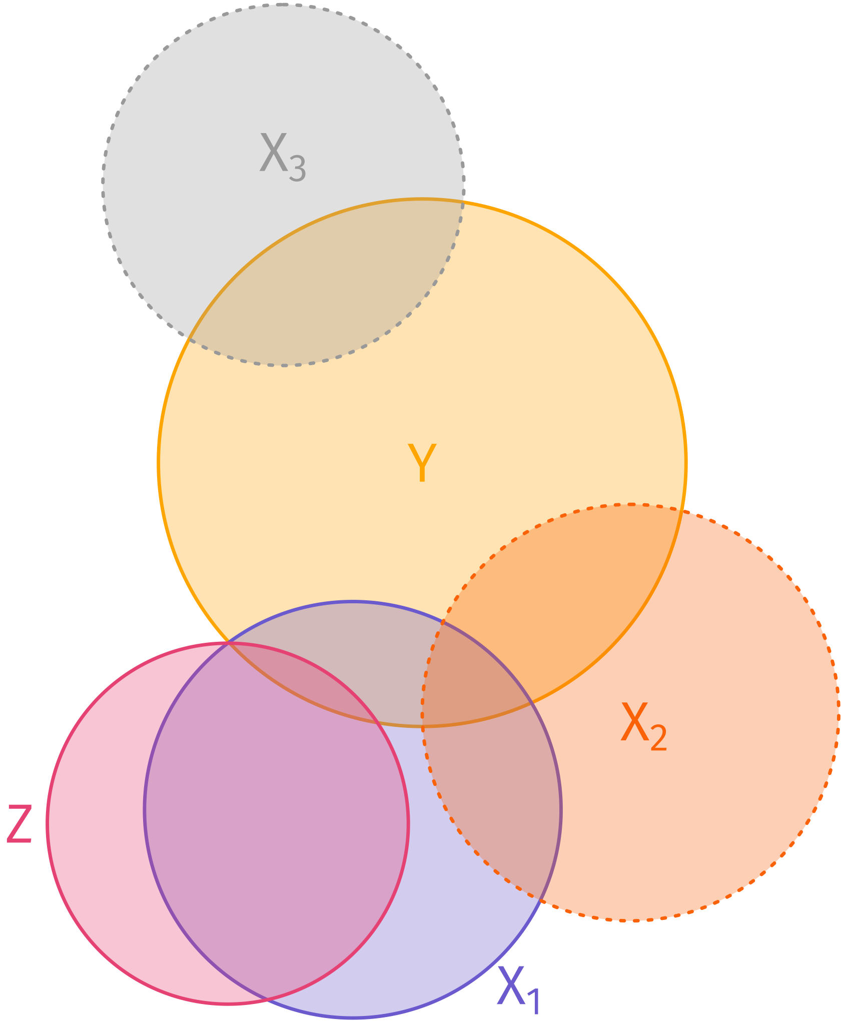


Figure 2

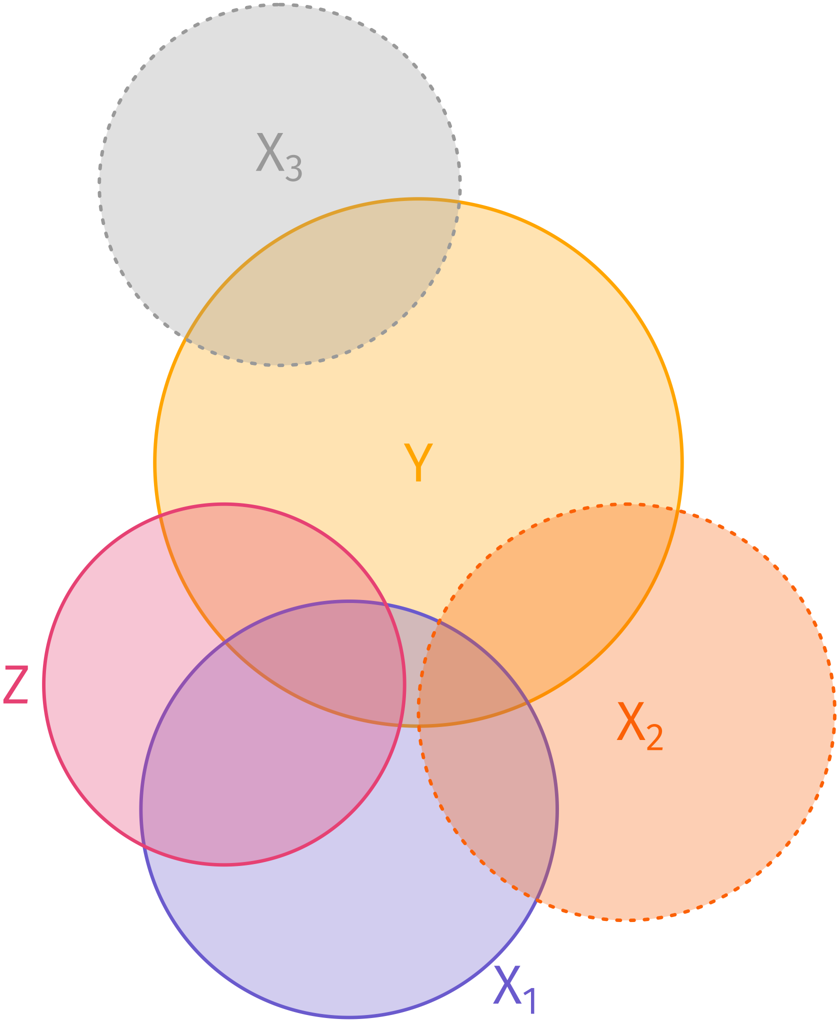


Figure 3

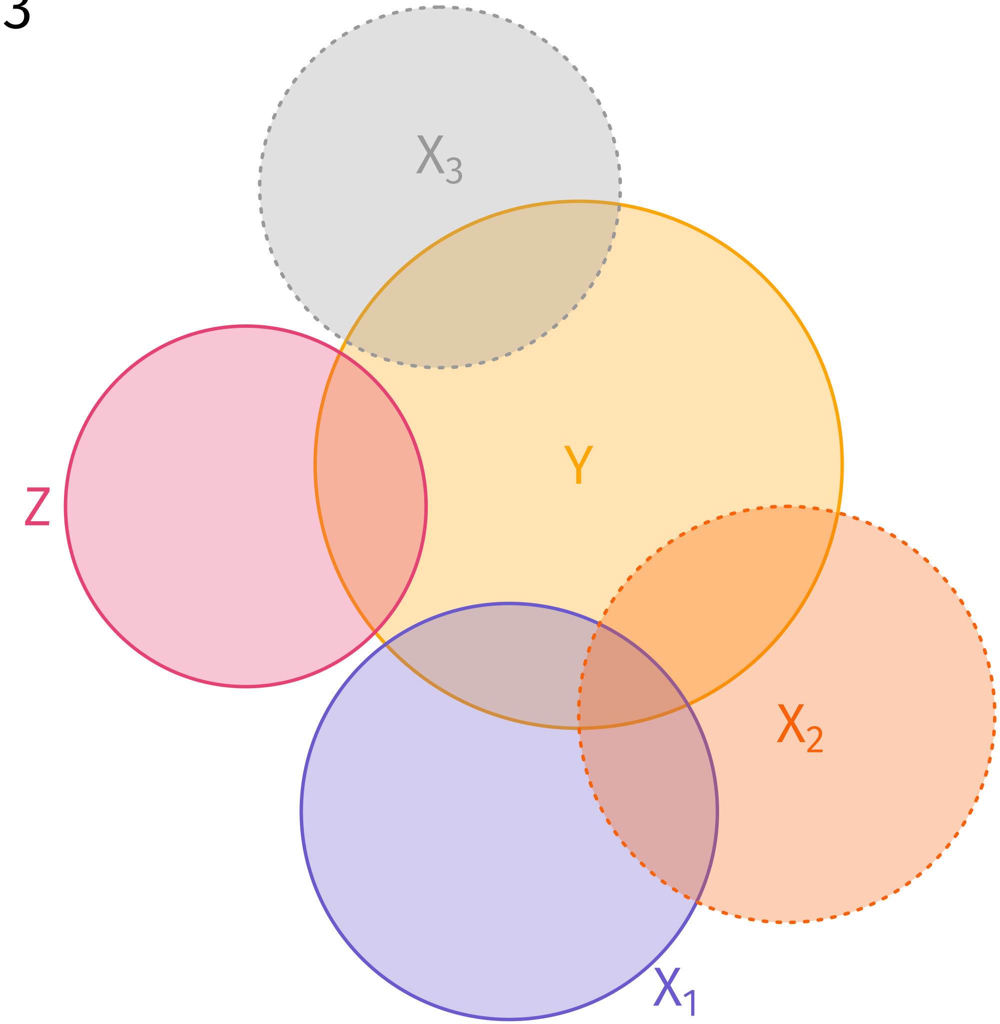
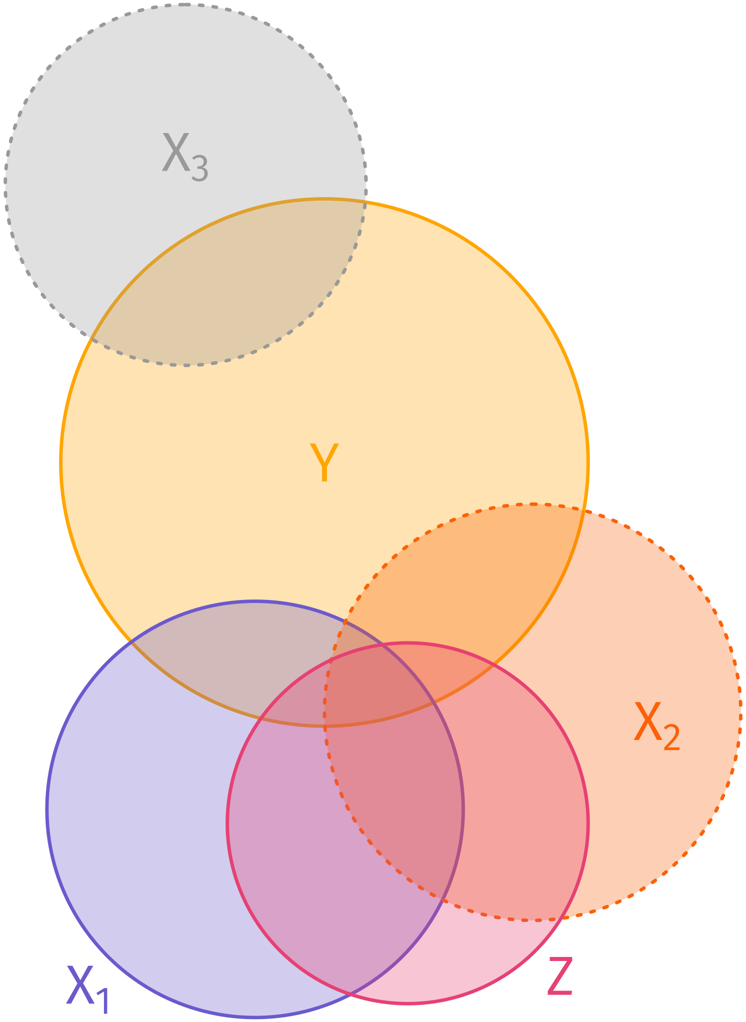


Figure 4



Venn diagram explanation

In these figures (Venn diagrams)

- Each circle illustrates a variable.
- Overlap gives the share of correlatation between two variables.
- Dotted borders denote *omitted* variables.

Take-aways

- Figure 1: **Valid instrument** (relevant; exogenous)
- Figure 2: **Invalid instrument** (relevant; not exogenous)
- Figure 3: **Invalid instrument** (not relevant; not exogenous)
- Figure 4: **Invalid instrument** (relevant; not exogenous)

Let's work an example in `R`.

Instrumental variables

Example in R

Back to our age-old battle to estimate the returns to education.

```
#> # A tibble: 722 x 4
#>     wage education education_dad education_mom
#>    <int>     <int>         <int>         <int>
#>  1   769        12             8             8
#>  2   808        18            14            14
#>  3   825        14            14            14
#>  4   650        12            12            12
#>  5   562        11            11             6
#>  6   600        10             8             8
#>  7  1154        15             5            14
#>  8  1000        12            11            12
#>  9   930        18            14            13
#> 10   900        15            12            12
#> # … with 712 more rows
```

Instrumental variables

Example in R

OLS for the returns to education with will likely (definitely) be biased...

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(Likely biased) OLS results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
Education	**58.594**	**6.439**	**9.10**	**<0.0001**

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but what if mother's education provides a valid instrument?

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This regression is known as the ***first stage***:

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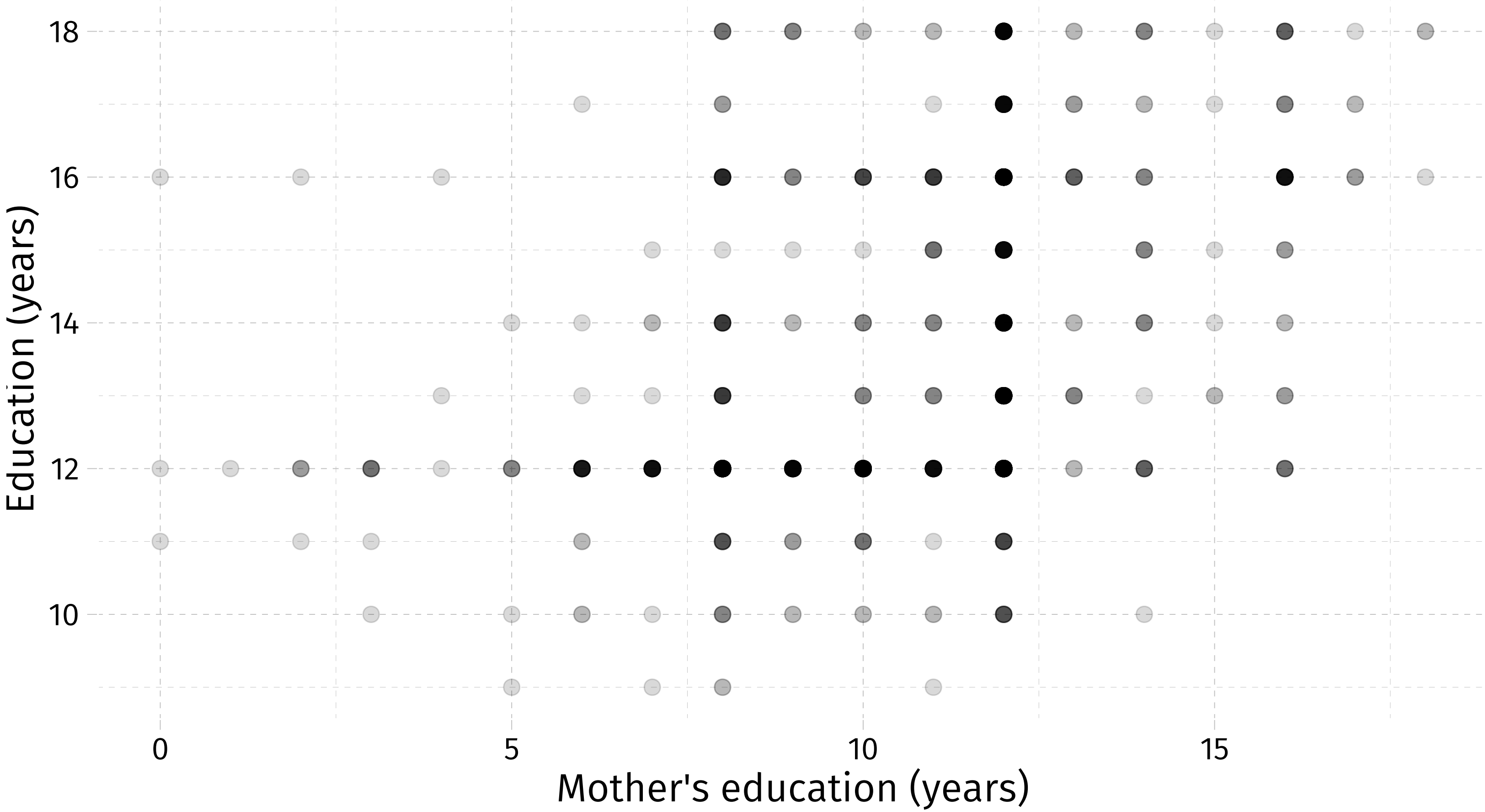
The *p*-value suggests a very strong relationship (very *relevant*).

Instrumental variables

Visualizing the first stage



Instrumental variables

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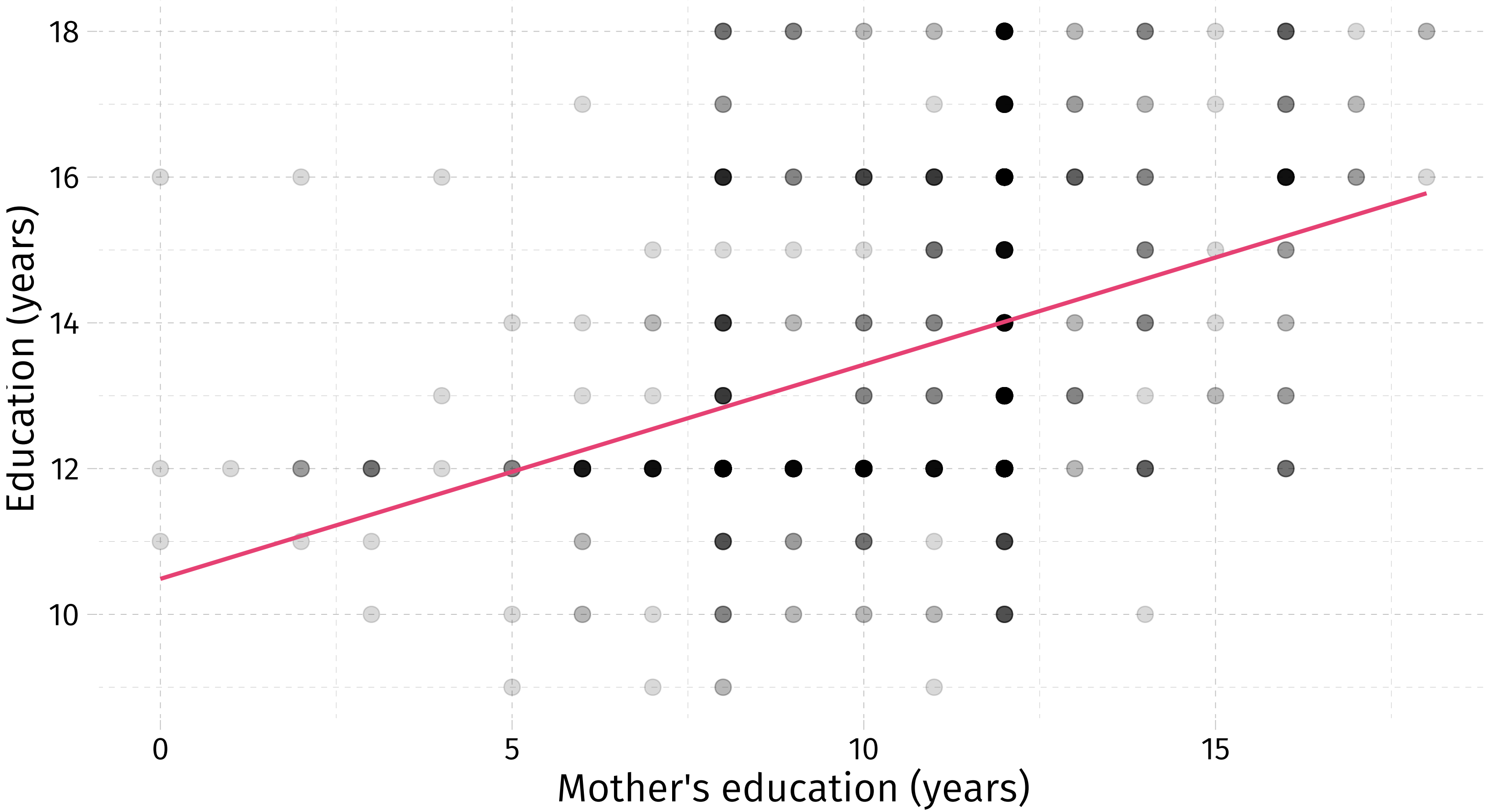

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Q: What does **exogeneity** mean in this case?

Instrumental variables

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A: We need two things

1. Mother's education (our instrument) must only affect earnings through education (our endogenous explanatory variable).
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Q: Does *mother's education* seem likely to satisfy exogeneity?

Instrumental variables

Example in R

Now, let's estimate the ***reduced form***:

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Term	Est.	S.E.	t stat.	p-Value
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Q_1: How do we interpret this estimated coefficient ($\hat{\pi}_1$)?

Q_2: If our instrument is *valid*, can we interpret these estimates as **causal**?

Example in R

So what is our IV-based estimate for the returns to education?

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Instrumental variables

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2. In the **first-stage equation**, we estimated $\hat{\gamma}_1 \approx 0.294$.

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$$\implies \hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{31.81}{0.294} \approx 108.2$$

Instrumental variables

Example in `R`

Alternative: Use the function `iv_robust()` from the `estimatr` package.

This new function `iv_robust` works very similar to our good friend `lm`:

```
iv_robust(y ~ x | z, data = dataset)
```

- `formula` Specify the regression followed by `|` and your instrument (`z`).
- `data` You still need a dataset.

Note: As you might guess by its name, `iv_robust` calculates heteroskedasticity-robust standard errors by default.

Instrumental variables

Example in R

In practice…

```
# Estimate our IV regression
iv_est ← iv_robust(wage ~ education | education_mom, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
Education	**108.214**	**16.810**	**6.44**	**<0.0001**

Instrumental variables

More

So now we know how to "do" instrumental variables

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So now we know how to "do" instrumental variables *when we have one endogenous variable and one exogenous variable.*

Instrumental variables

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So now we know how to "do" instrumental variables *when we have one endogenous variable and one exogenous variable.*

1. Estimate the reduced form (regress outcome var. on instrument).
2. Estimate the first stage (regress expl. var. on instrument).
3. Calculate the IV estimate using the estimates from (1) and (2).

Our magical instrument isolates the exogenous variation in our endogenous variable.

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Q: What if we want more? (*E.g.*, more instruments or endog. variables)
A: ~~Too bad.~~ Extend IV to **two-stage least squares (2SLS)**.

Two-stage least squares

Two-stage least squares

Intro

The intuition and insights of IV carry over into two-stage least squares.

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$$
\begin{aligned}
\text{Endogenous model} \qquad \text{Outcome}_i &= \beta_0 + \beta_1 (\text{Endog. var.})_i + u_i \\
\text{First stage} \qquad (\text{Endog. var.})_i &= \pi_0 + \pi_1 \text{Instrument}_i + v_i \\
\text{Second stage} \qquad \text{Outcome}_i &= \delta_0 + \delta_1 \widehat{(\text{Endog. var.})}_i + \varepsilon_i \\
\text{Reduced form} \qquad \text{Outcome}_i &= \pi_0 + \pi_1 \text{Instrument}_i + w_i
\end{aligned}
$$

where $\widehat{(\text{Endog. var.})}_i$ denotes the predicted values (*fitted values*) from the first-stage regression.

Two-stage least squares

Intro

Two-stage least squares is very flexible—we include other controls, additional endogenous variables, *and* have multiple instruments.

But don't get too distracted by this fancy flexiblity.

We still need **valid** instruments.

Two-stage least squares

In R

Back to our *returns to education* example.

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + u_i$$

Imagine that mother's *and* father's education are both valid instruments.

Then our **first-stage regression** is

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's education})_i + \gamma_2 (\text{Father's education})_i + v_i$$

which we can estimate via OLS.

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Q: Why?

Two-stage least squares

In R

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's education})_i + \gamma_2 (\text{Father's education})_i + v_i$$

```
stage1 <- lm(education ~ education_mom + education_dad, wage_df)
```

First-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	9.845	0.305	32.31	<0.0001
Mother's Education	**0.149**	**0.032**	**4.62**	**<0.0001**
Father's Education	**0.216**	**0.028**	**7.84**	**<0.0001**

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Our instruments each appear to be *relevant*.

Formally, we should jointly test them (*e.g.*, F test).

Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

$$\widehat{\text{Education}}_i = \widehat{\gamma}_0 + \widehat{\gamma}_1(\text{Mother's education})_i + \widehat{\gamma}_2(\text{Father's education})_i$$

Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

$$\widehat{\text{Education}}_i = \widehat{\gamma}_0 + \widehat{\gamma}_1 (\text{Mother's education})_i + \widehat{\gamma}_2 (\text{Father's education})_i$$

```r
# Add fitted values from first stage
wage_df$education_hat <- stage1$fitted.values
```

Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

$$\widehat{\text{Education}}_i = \widehat{\gamma}_0 + \widehat{\gamma}_1 (\text{Mother's education})_i + \widehat{\gamma}_2 (\text{Father's education})_i$$

```r
# Add fitted values from first stage
wage_df$education_hat <- stage1$fitted.values
```

Now we use OLS (again) to estimate the **second-stage regression**

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

Two-stage least squares

In R

$$\color{orange}{\text{Wage}_i} = \delta_0 + \delta_1 \color{purple}{\widehat{\text{Education}}_i} + \varepsilon_i$$

```
stage2 <- lm(wage ~ education_hat, wage_df)
```

Second-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
Fitted Education	**104.789**	**14.462**	**7.25**	**<0.0001**

Ordinary least squares

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
Education	**58.594**	**6.439**	**9.10**	**<0.0001**

Instrumental variables

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
Education	**108.214**	**16.810**	**6.44**	**<0.0001**

Two-stage least squares w/ two instruments

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
Education	**104.789**	**14.462**	**7.25**	**<0.0001**

Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + ⋯ | z1 + z2 + ⋯, data)
```

Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + ⋯ | z1 + z2 + ⋯, data)
```

In our case, we have

- one explanatory variable (`x`) (education)
- two instruments (`z`) (parents' educations)

```
iv_robust(wage ~ education | education_mom + education_dad, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	199.946	-2.27	0.0233
Education, fitted	**104.789**	**14.852**	**7.06**	**<0.0001**

Two-stage least squares

There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

Two-stage least squares

There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

Common applications

- **General causal inference** for observational data (as we've seen).
- **Experiments:** Randomize a treatment that affects an endog. variable.
- **Measurement error:** Regress noisy x_1 on noisy x_2 to capture signal.
- **Simultaneous relationships** (*e.g.*, p and q from supply and demand).

However, in any 2SLS/IV setting, you need to mind the requirements for **valid instruments**—exogeneity and relevance.

Table of contents

Admin

Instrumental variables