Causality

EC 421, Set 10

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28 February 2019

Prologue

Schedule

Last Time

- Autocorrelation, nonstationarity, 'in-class' analysis
- **Follow up:** `EC422` (time series) is only offered in the winter. 😢
- **Follow up:** `EC410` (computational economics) in the spring! 😃
- **Follow up:** `R` is mainly written in `C`, `R`, and `Fortran`.

Today

- Return to our in-class examples
- Causality

Upcoming

Assignment due Sunday. Another one coming soon.

Problems and strategies

Step 1: Define the problem.

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The **true data-generating process** (DGP).

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A few options:

1. Find the combination of variables that **maximize R^2** or **adjusted R^2**.
2. First **include all** variables. Keep statistically **significant variables**.
3. Iterate with (2.): **Drop non-significant variables** until nothing changes.
4. **Add variables one by one**. Keep statistically **significant variables**.
5. **Plot** variables' (or residuals') relationships with y.

```r
# Load the data
fun_df <- read_csv("fun_data.csv")
# Separate into two datasets
y1_df <- fun_df %>% select(-y2)
y2_df <- fun_df %>% select(-y1)
# Peak at the data
y1_df
```

```
#> # A tibble: 100 x 10
#>          y1     x1    x2     x3    x4     x5    x6       x7    x8    x9
#>       <dbl>  <dbl> <dbl>  <dbl> <dbl>  <dbl> <dbl>    <dbl> <dbl> <dbl>
#>  1   3.08   -0.777 0.405  1.23  0.762 -0.232  1.17  0.111       1 1.98
#>  2   6.04    0.473 1.59   0.584 1.53   0.349  1.52 -0.00994     2 0.511
#>  3   9.57    2.30  3.52  -0.976 3.32   0.581  1.50  0.974       3 0.936
#>  4  11.4     2.46  5.33  -1.77  4.64  -0.576  1.92  2.53        4 2.88
#>  5  -0.0319  0.313 2.09  -2.59  1.37  -0.717  3.76  2.14        5 2.20
#>  6   5.21    1.37  1.23   2.34  2.21  -1.40   3.55  1.17        6 1.83
#>  7   7.97    1.73  3.46   0.584 2.24  -1.31   3.77  1.92        7 1.75
#>  8  -5.17    2.60  4.09  -4.15  4.13  -2.57   4.60  0.886       8 1.14
#>  9   1.57    0.877 3.96   2.08  1.42  -2.89   3.68  1.32        9 2.23
#> 10   3.97   -0.197 0.875 -0.760 0.697 -1.92   1.90  1.85       10 1.90
#> # … with 90 more rows
```

R showcase

`gather`ing data

Let's plot y_1 against the nine potential explanatory variables, x_1 to x_9.

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We'll use two new functions to streamline this process.

- `gather()` (from `dplyr`): *Stacks* variables (names and values).
- `facet_wrap()`: Creates multiple plots grouped by a variable.

`gather`ing data

Example: `gather` all variables in our dataset.

```
data.frame(w = 0:1, x = 2:3, y = 4:5, z = 6:7) %>%
  gather(key = "var", value = "value")
```

```
#>   var value
#> 1   w     0
#> 2   w     1
#> 3   x     2
#> 4   x     3
#> 5   y     4
#> 6   y     5
#> 7   z     6
#> 8   z     7
```

`gather`ing data

Example: `gather` all variables in our dataset except `w`.

```
data.frame(w = 0:1, x = 2:3, y = 4:5, z = 6:7) %>%
  gather(-w, key = "var", value = "value")
```

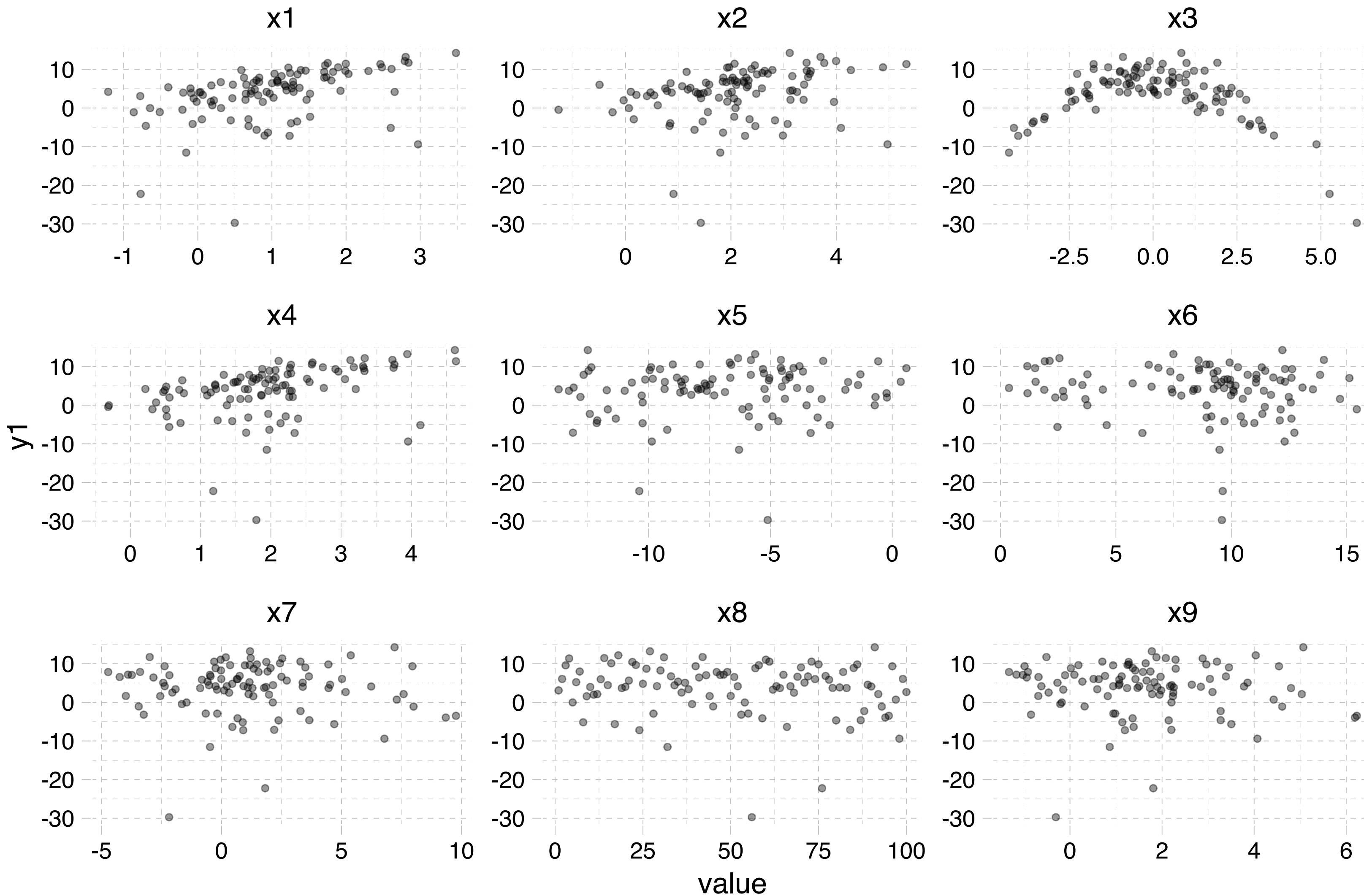
```
#>   w var value
#> 1 0   x     2
#> 2 1   x     3
#> 3 0   y     4
#> 4 1   y     5
#> 5 0   z     6
#> 6 1   z     7
```

`gather`ing data

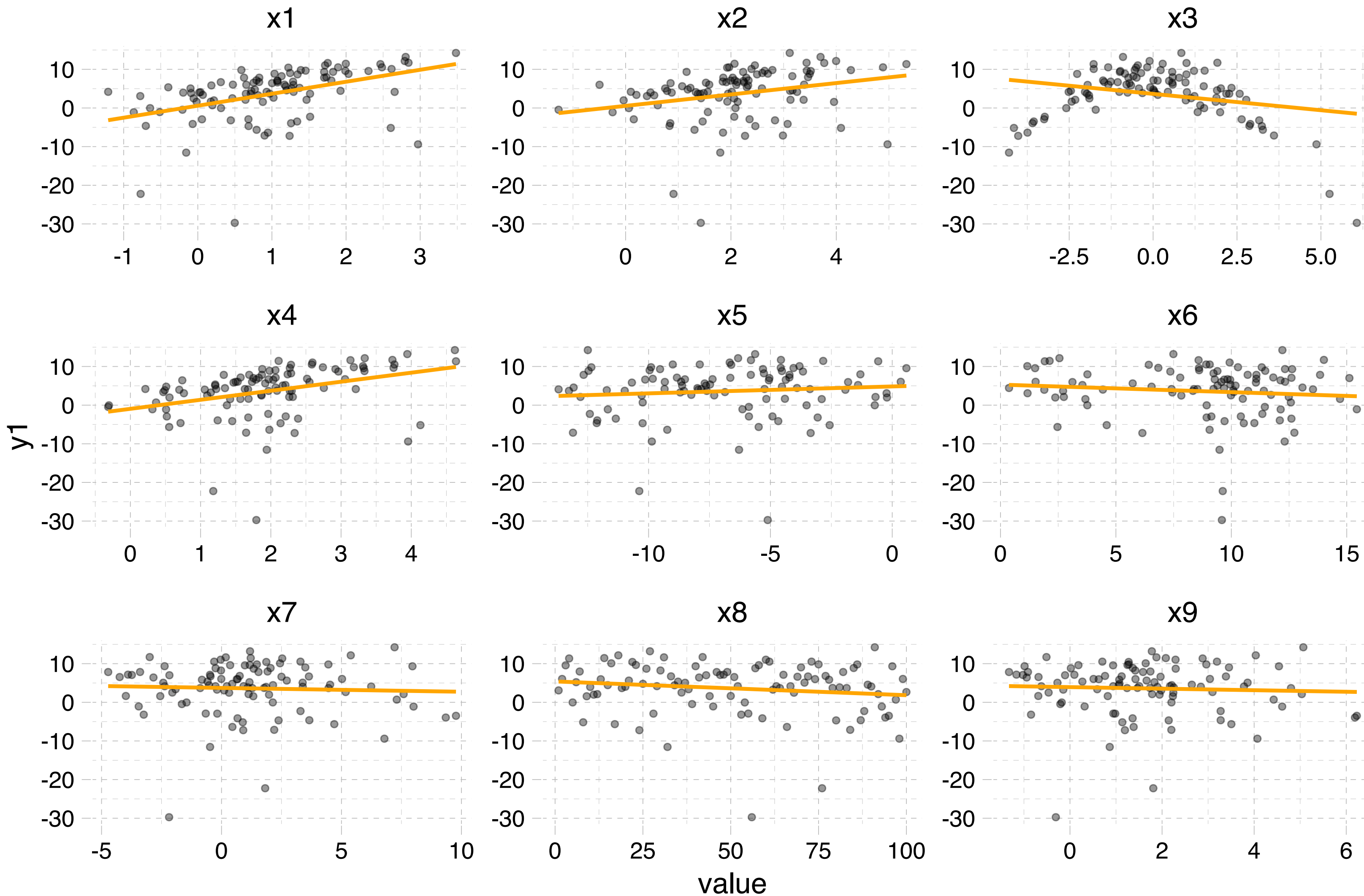
Adding these new functions to our previous `ggplot2` work...

```
y1_df %>% gather(-y1, key = "var", value = "value") %>%
  ggplot(aes(x = value, y = y1)) +
  geom_point(alpha = 0.4, size = 1.5) +
  facet_wrap(~ var, scales = "free") +
  theme_pander(base_size = 16)
```

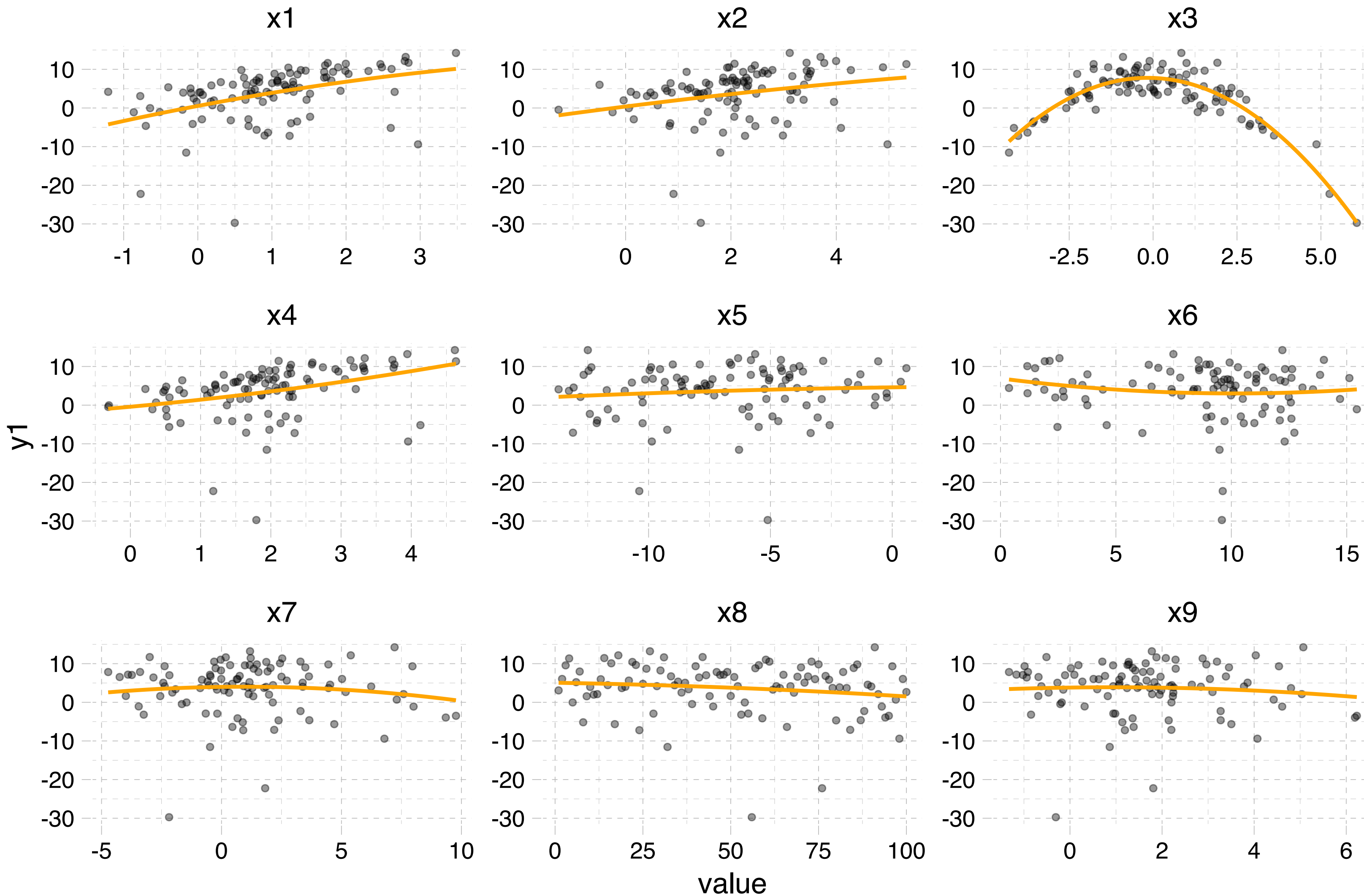
Plot: y_1 against x_1 through x_9



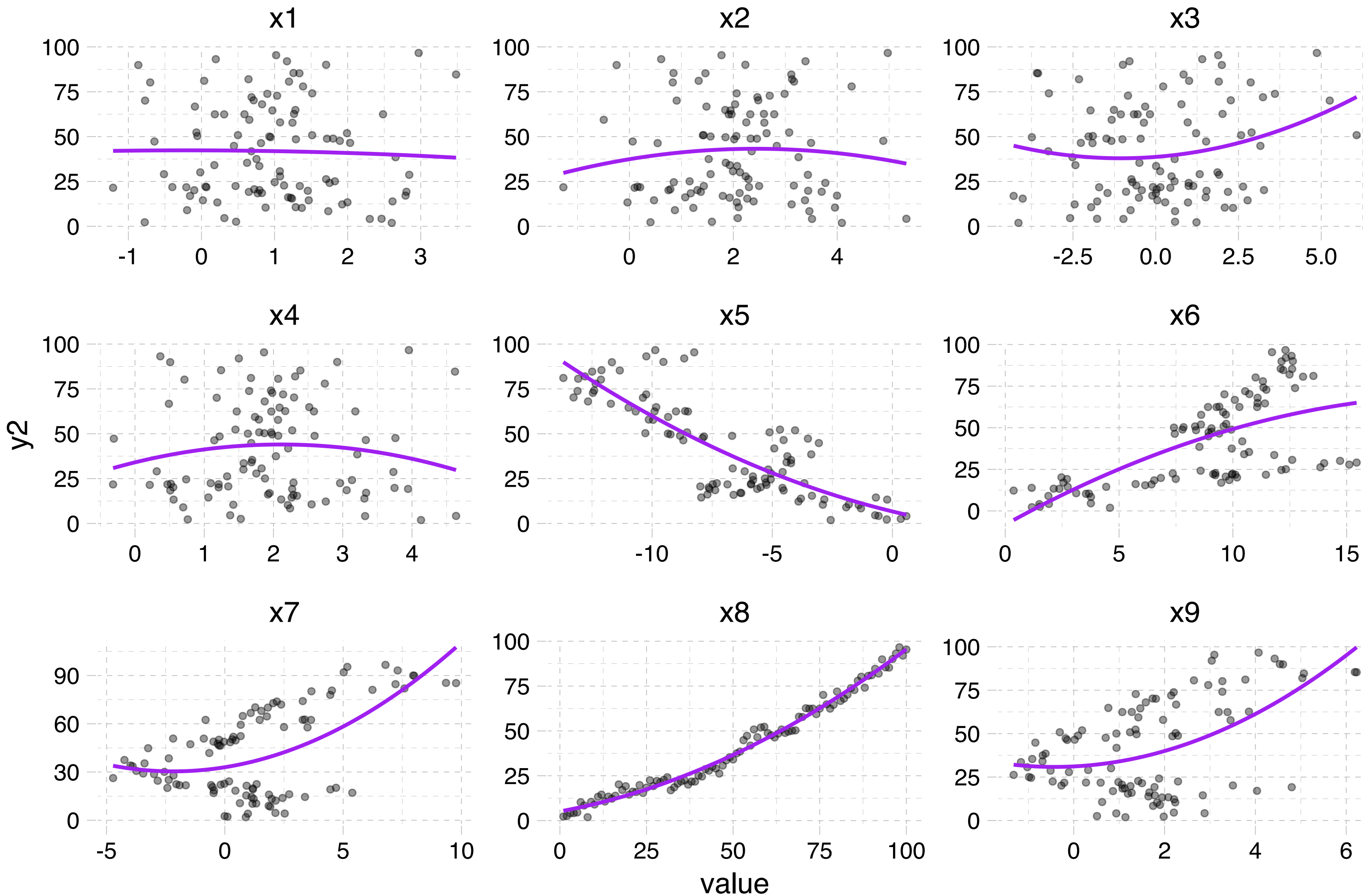
Simple linear regressions: y_1 against x_1 through x_9



Linear regressions with quadratic RHS: y_1 against x_1 through x_9



Linear regressions with quadratic RHS: y_2 against x_1 through x_9



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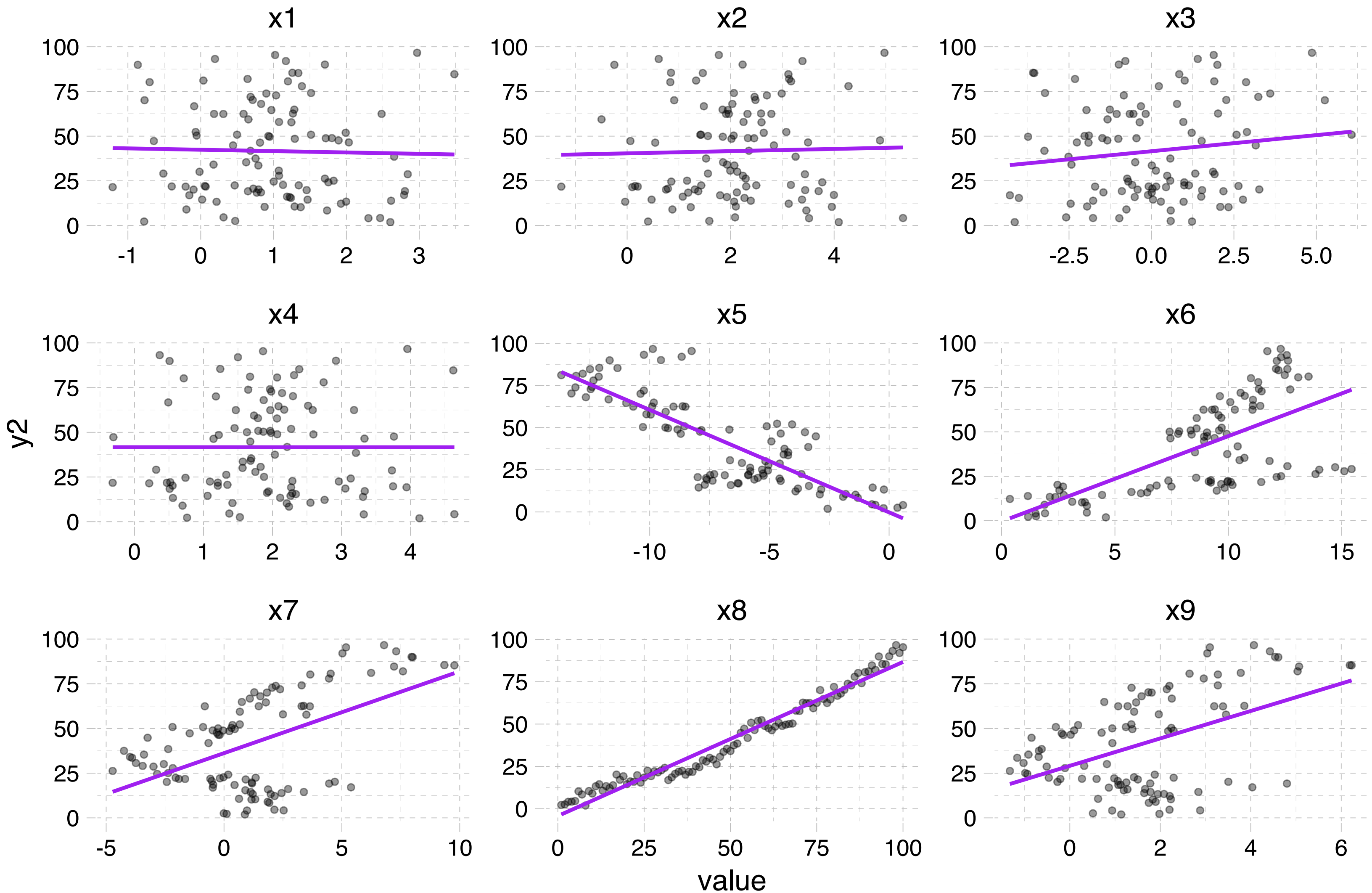

Searching for the unknown model

Results

Your responses: Percentage who said `TRUE` (29 responses)

	X1	X2	X3	X4	X5	X6	X7	X8	X9
y1	78.6	7.1	60.7	39.3	28.6	28.6	17.9	17.9	25.0
y2	46.4	50.0	64.3	10.7	75.0	57.1	75.0	53.6	46.4

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Truth: The true data-generating processes

$$y_1 = 3 + x_1 - x_3^2 + 2x_4 + u$$
$$y_2 = 1 + x_3 + x_5 + x_7 + v$$

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Q: Is it worse include an incorrect variable or exlcude a correct variable?

Causality

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2. **Causal estimation:**[†] Estimate the actual data-generating process—learning about the true, population model that explains how y changes when we change x_j—focuses on β_j. Accuracy of $\hat{y}$ is not important.

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For the rest of the term, we will focus on **causally estimating** β_j.

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Causality requires us to **hold all else constant** (*ceterus paribus*).

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- What causes some countries to grow and others to decline?
- What caused President Trump's 2016 election?
- How does the number of police officers affect crime?
- What is the effect of better air quality on test scores?
- Do longer prison sentences decrease crime?
- How did cannabis legalization affect mental health/opioid addition?

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New saying:

> Correlation plus exogeneity is causation.

Let's work through a few examples.

Causation

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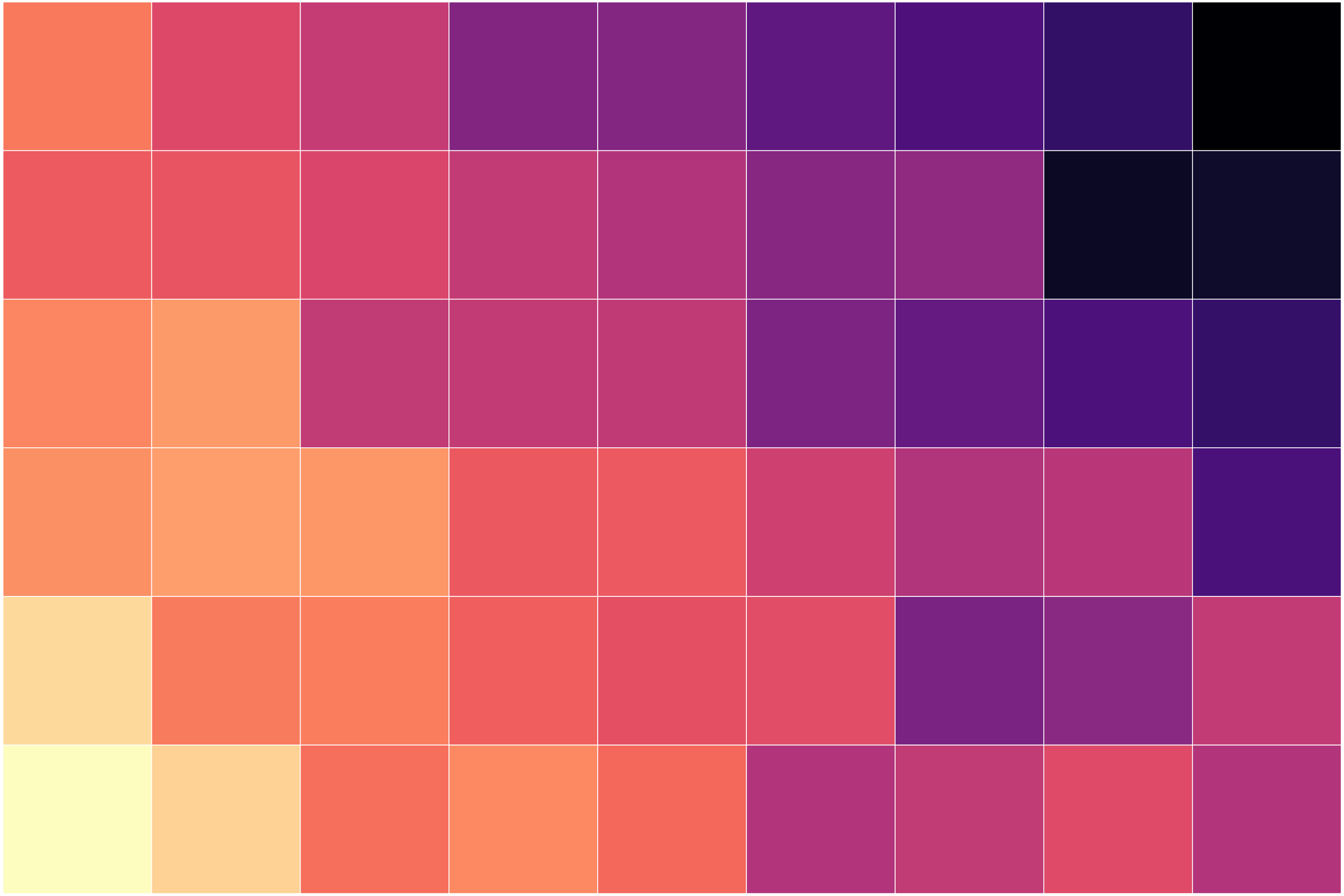
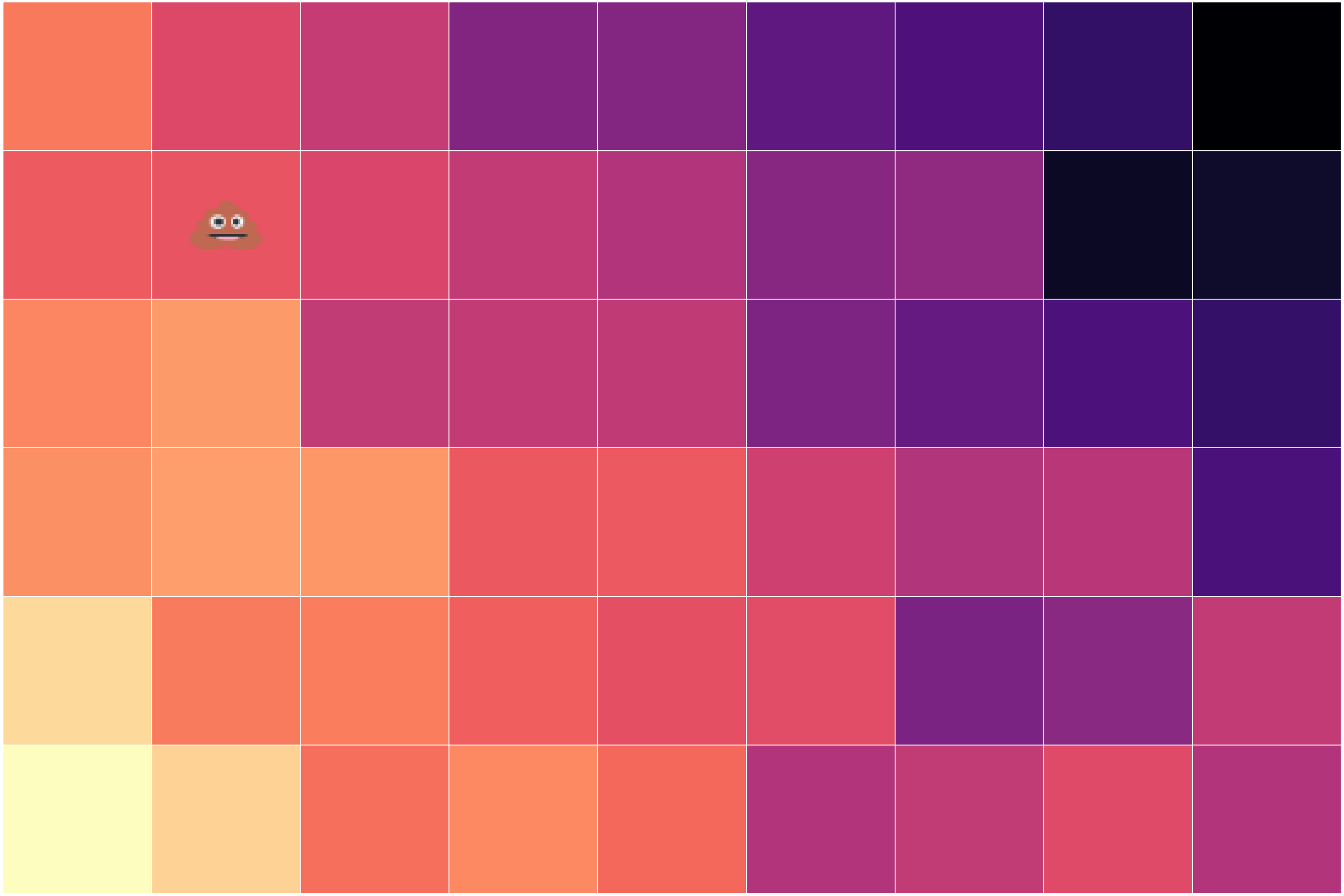
54 equal-sized plots

01	02	03	04	05	06	07	08	09
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54

54 equal-sized plots of varying quality



54 equal-sized plots of varying quality plus randomly assigned treatment



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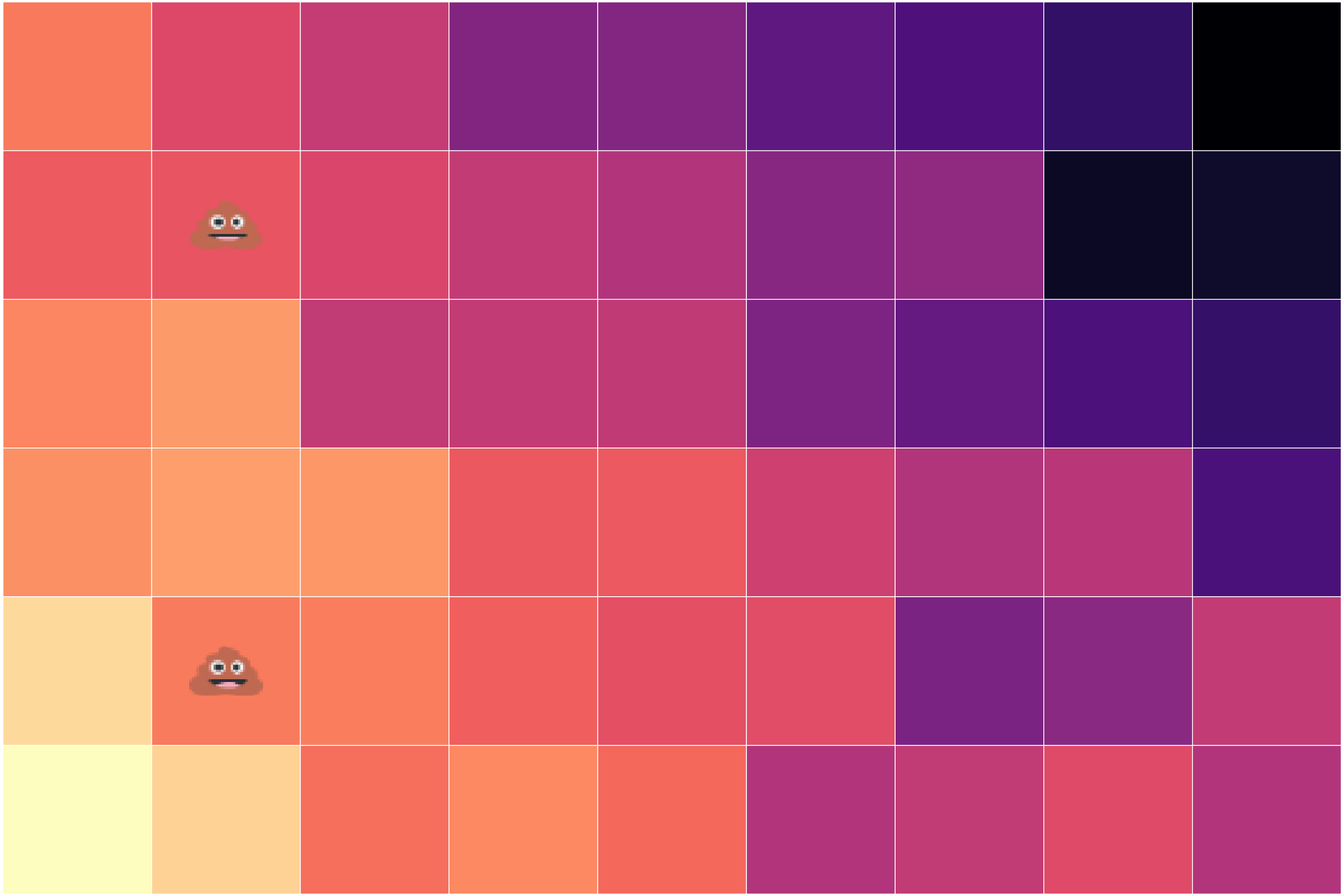

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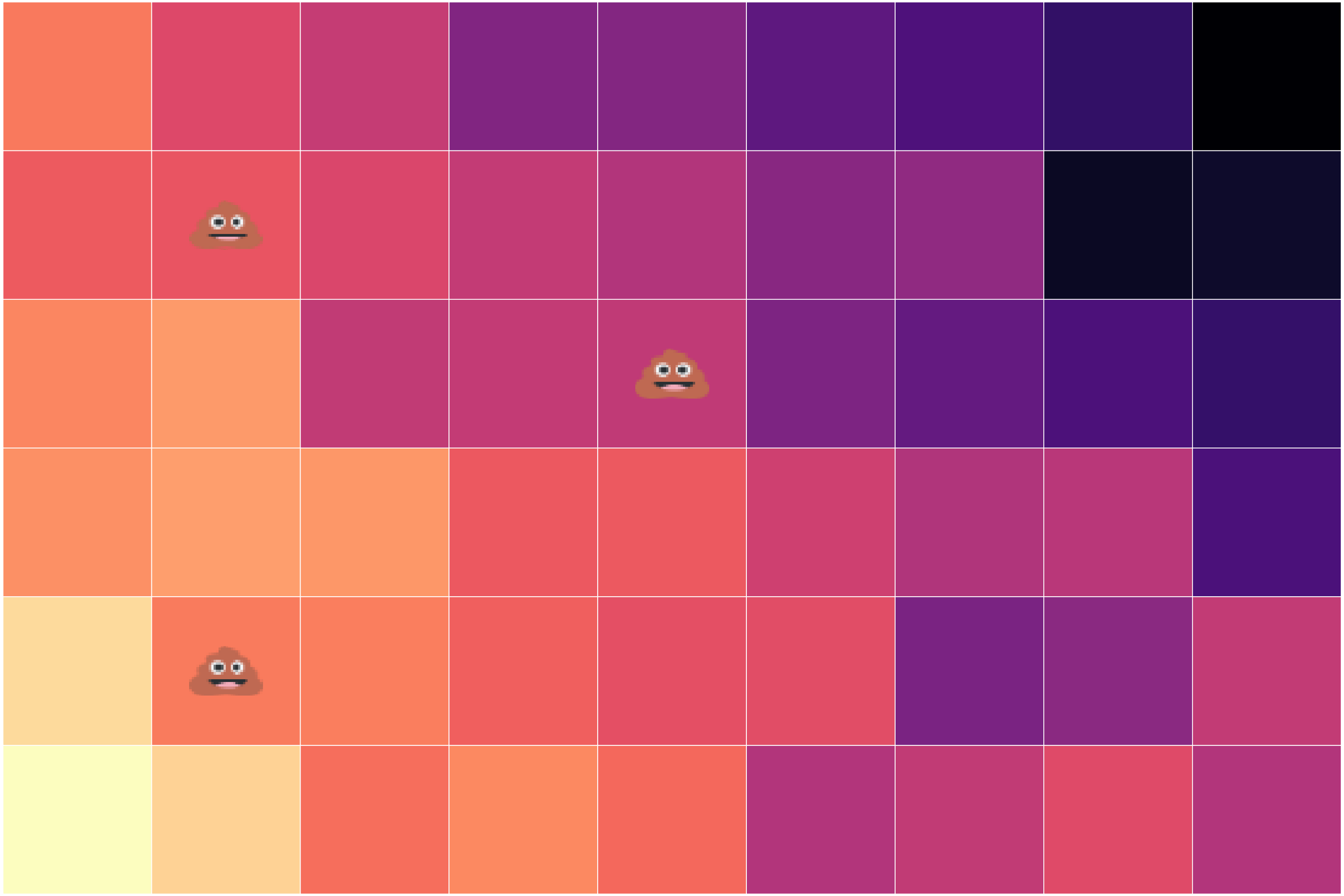

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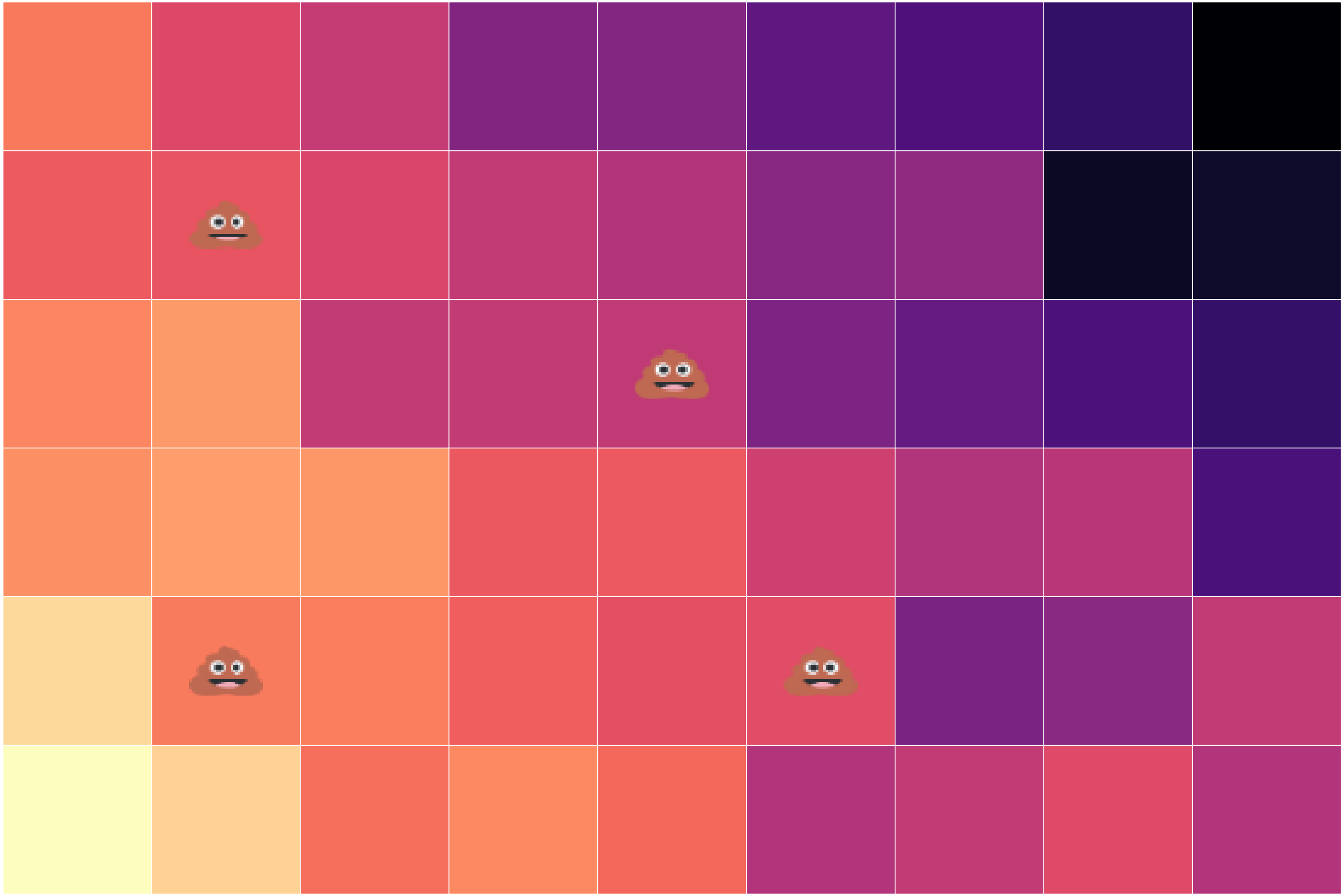

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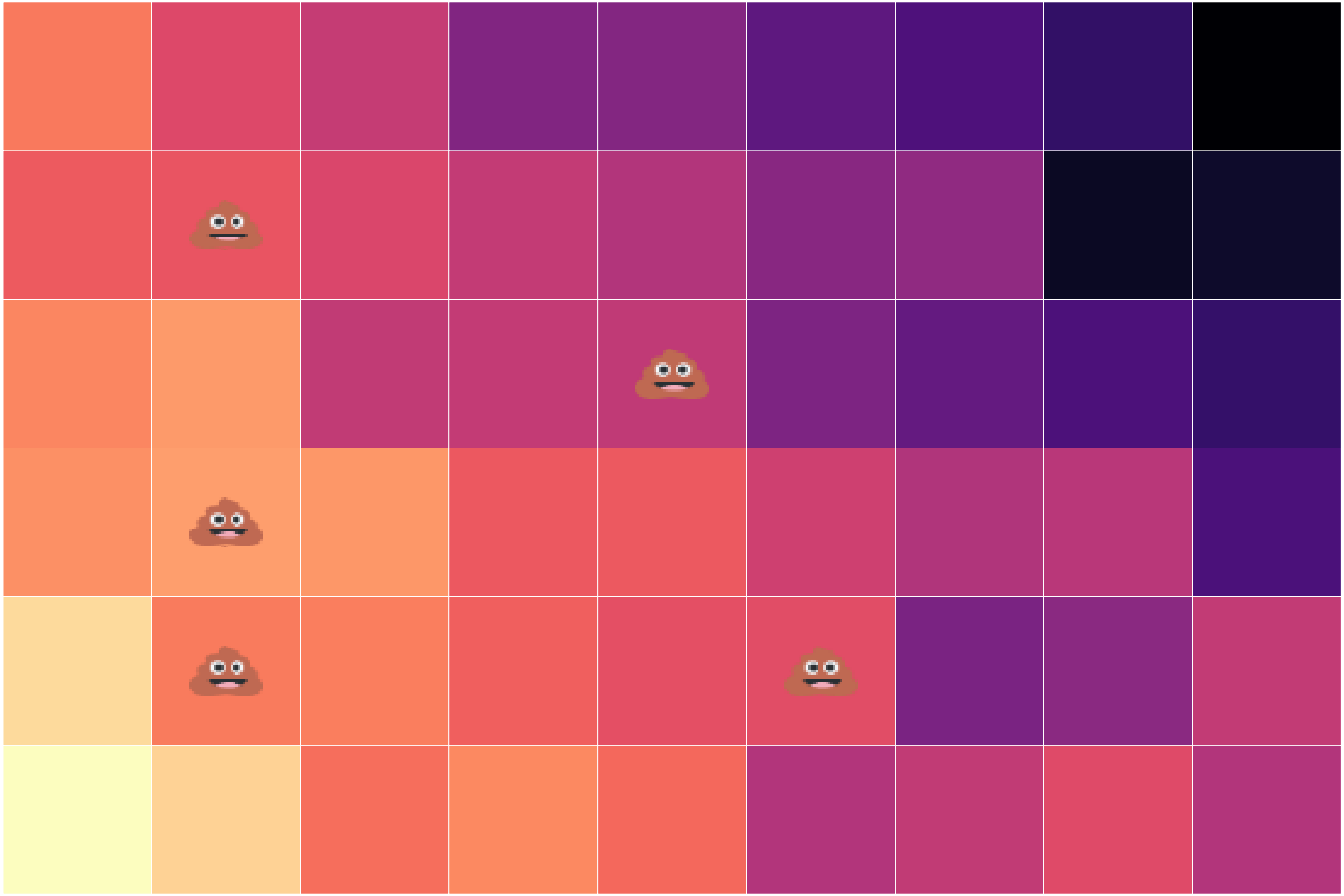

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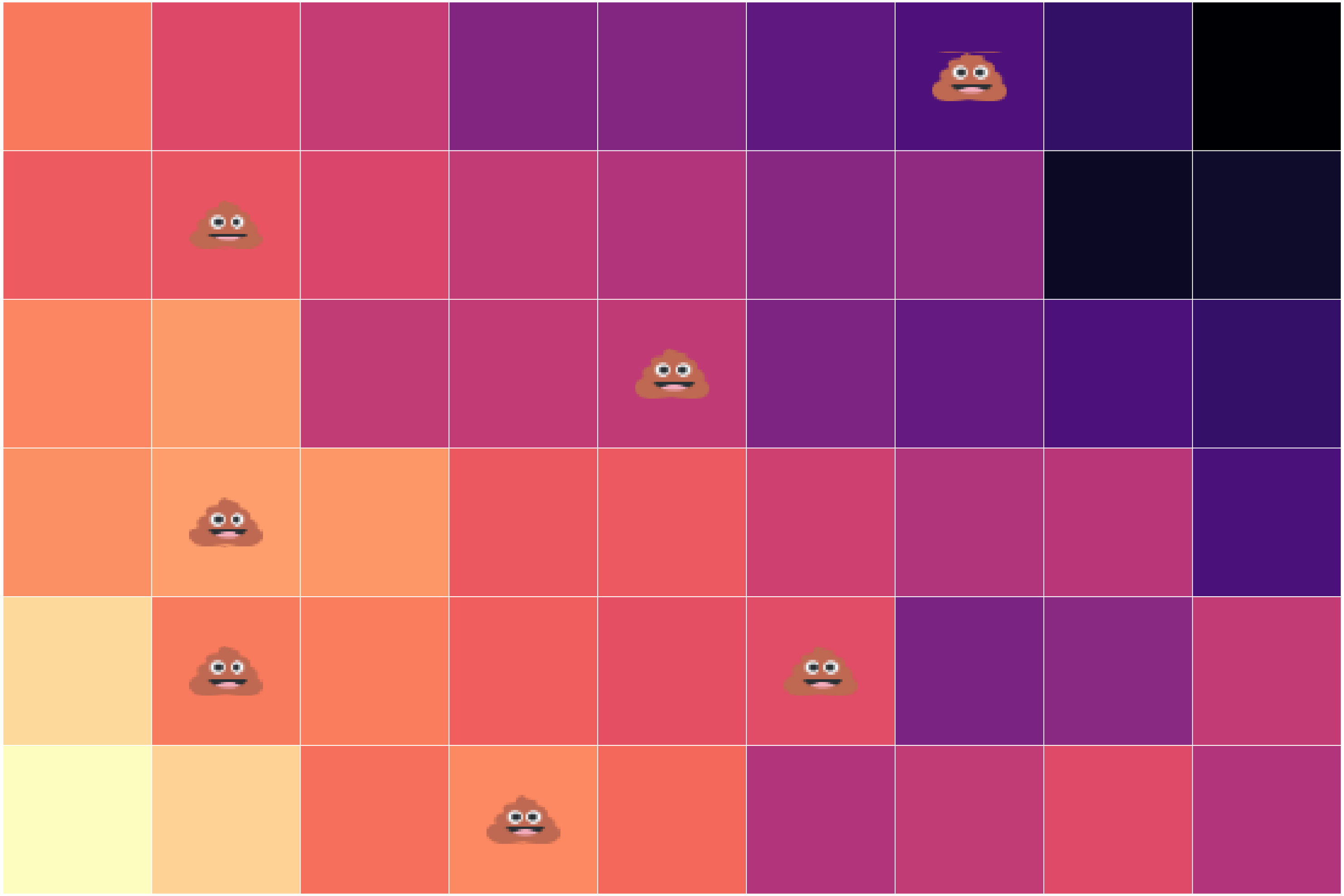

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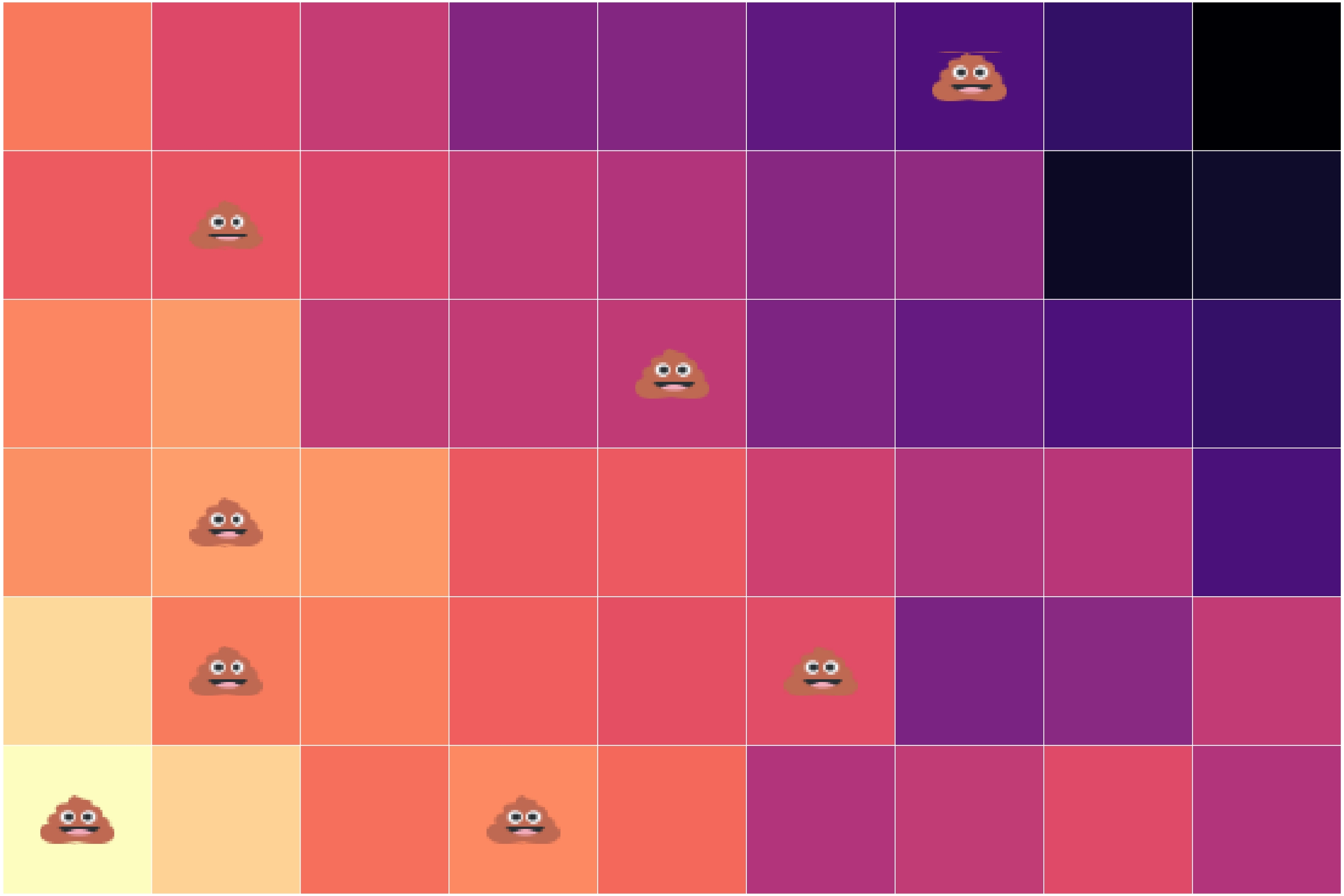

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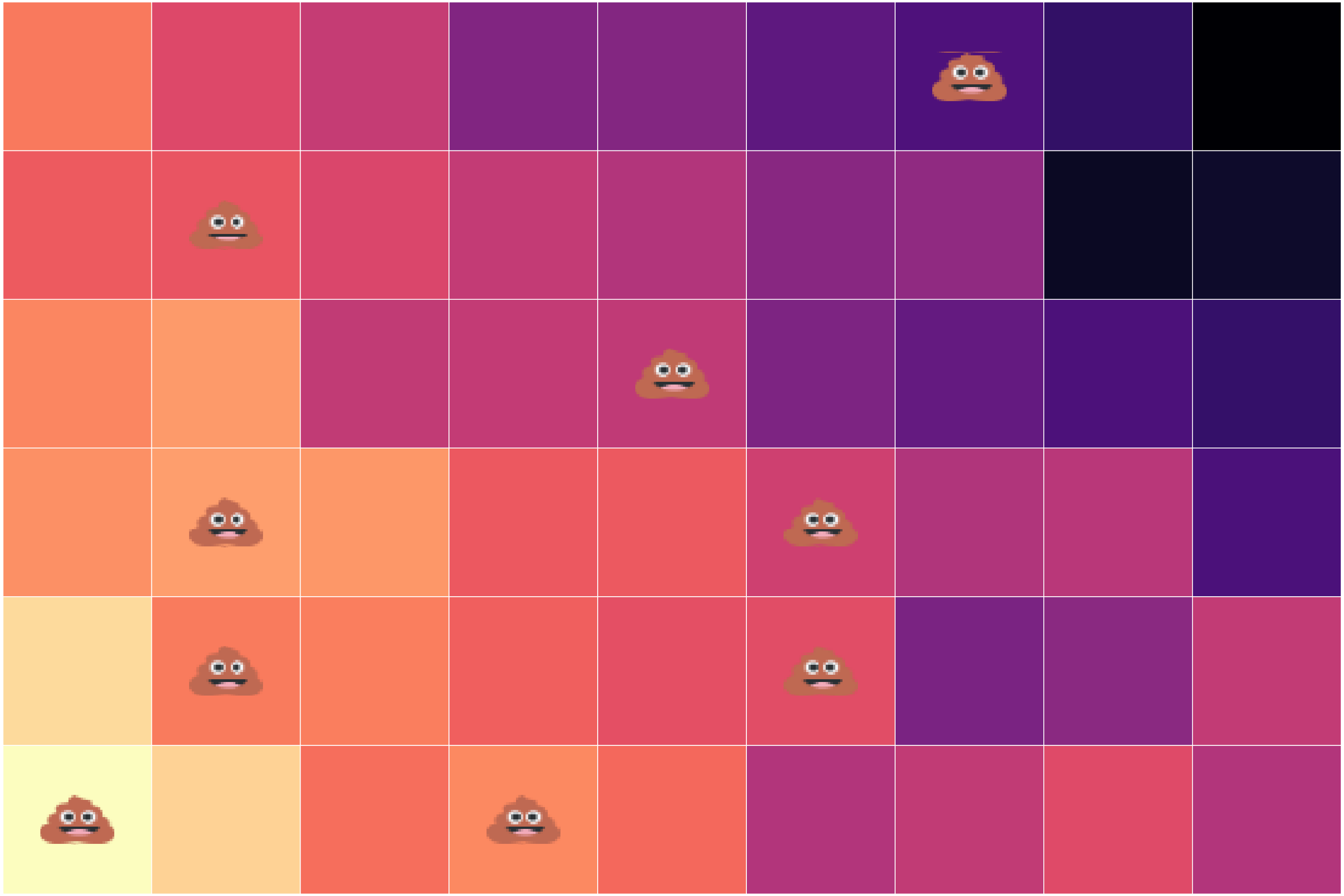

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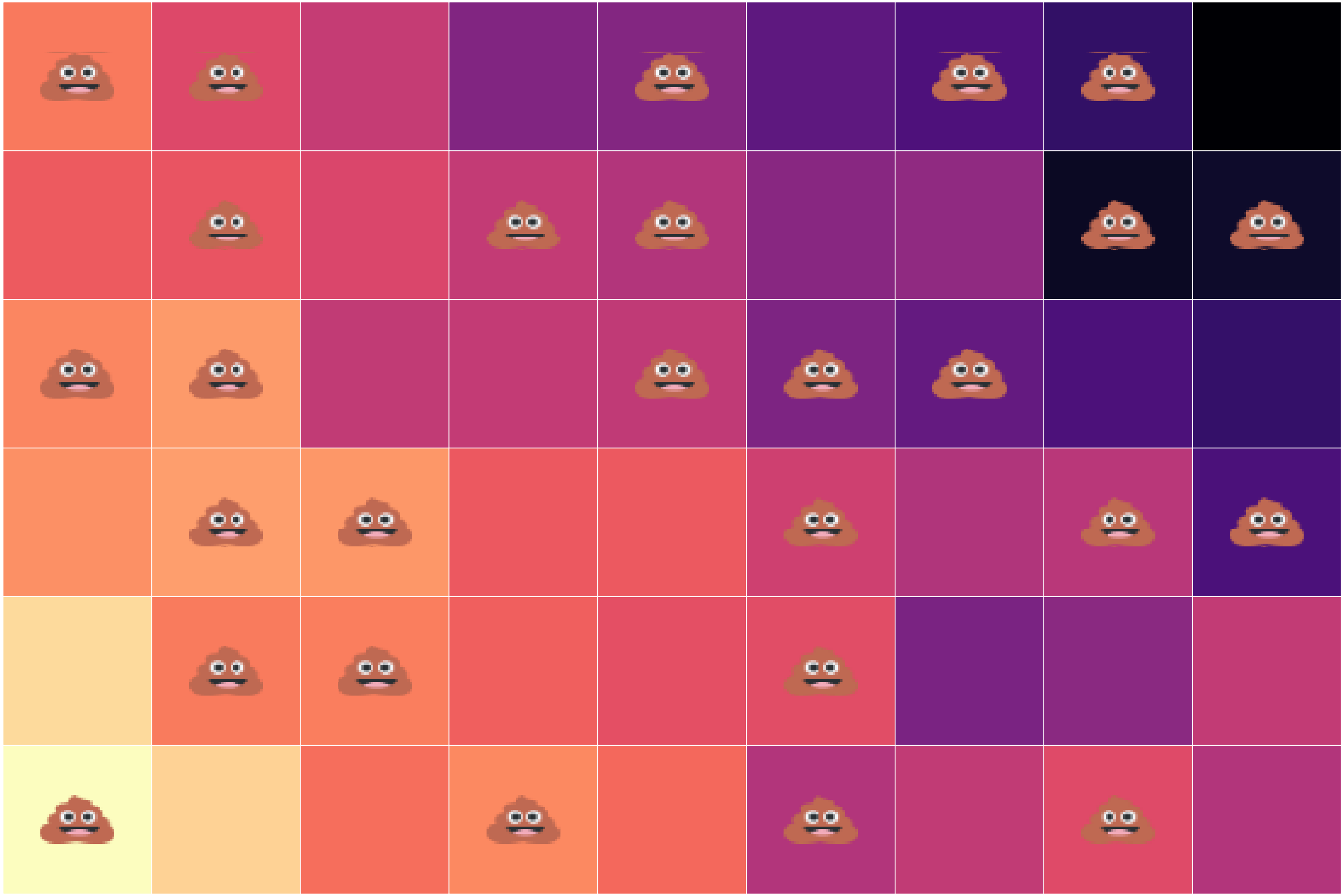

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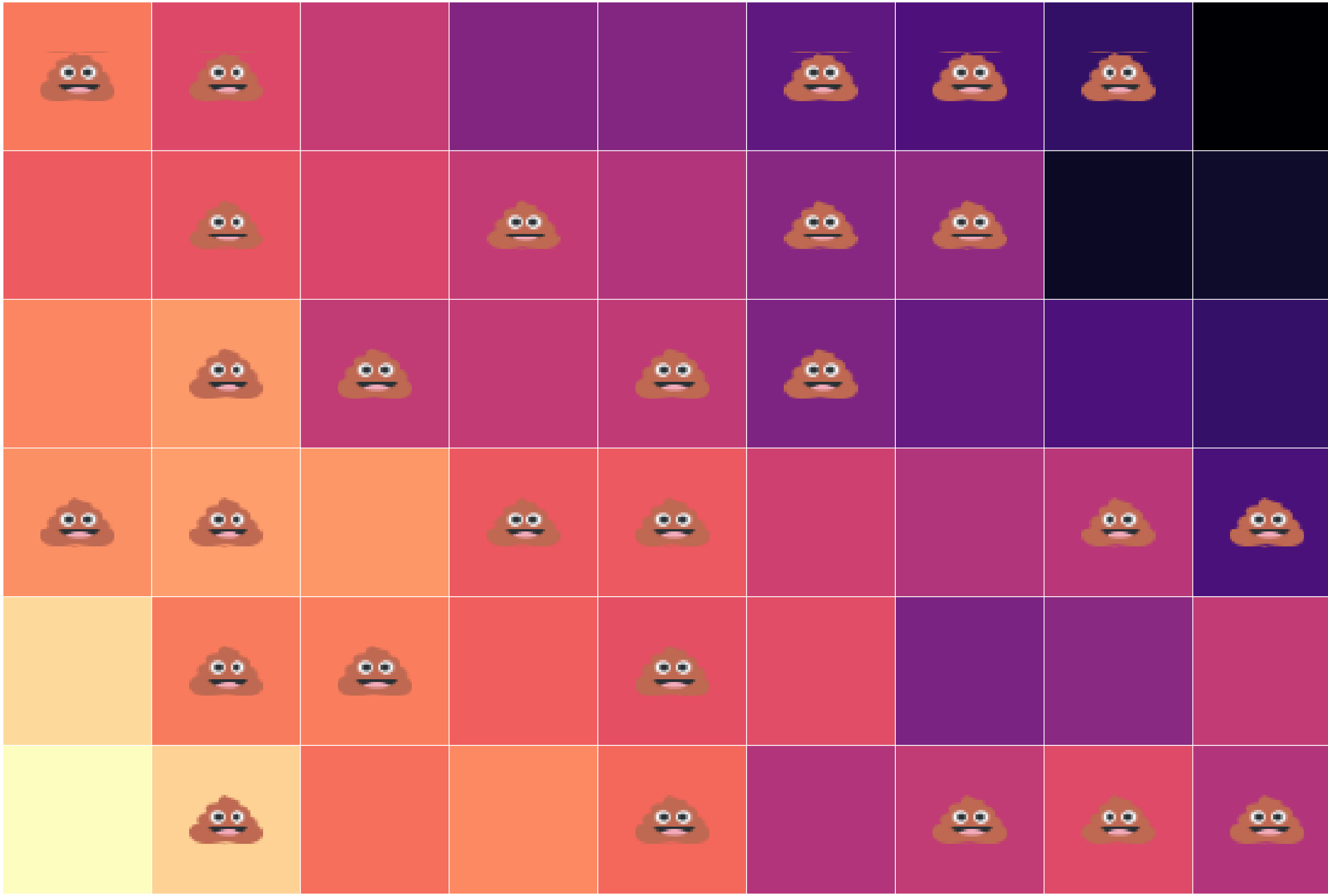

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We can estimate the **causal effect** of fertilizer on crop yield by comparing the average yield in the treatment group (💩) with the control group (no 💩).

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Q: Should we expect (1) to satisfy exogeneity? Why?

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Alternatively, we can use the regression

$$\text{Yield}_i = \beta_0 + \beta_1 \text{Trt}_i + u_i \tag{1}$$

where Trt_i is a binary variable (=1 if plot i received the fertilizer treatment).

Q: Should we expect (1) to satisfy exogeneity? Why?

A: On average, **randomly assigning treatment should balance** trt. and control across the other dimensions that affect yield (soil, slope, water).

Causality

Example: Returns to education

Labor economists, policy makers, parents, and students are all interested in the (monetary) *return to education.*

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Thought experiment:

- Randomly select an individual.
- Give her an additional year of education.
- How much do her earnings increase?

This change in earnings gives the **causal effect** of education on earnings.

Causality

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The point (2) above also illustrates the difficulty in learning about educations while *holding all else constant*.

Many important variables have the same challenge—gender, race, income.

Causality

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- Admissions cutoffs
- Lottery enrollment and/or capacity constraints

Causality

Real-world experiments

Both examples consider **real experiments** that isolate causal effects.

Characteristics

- Feasible—we can actually (potentially) run the experiment.
- Compare individuals randomized into treatment against individuals randomized into control.
- Require "good" randomization to get *all else equal* (exogeneity).

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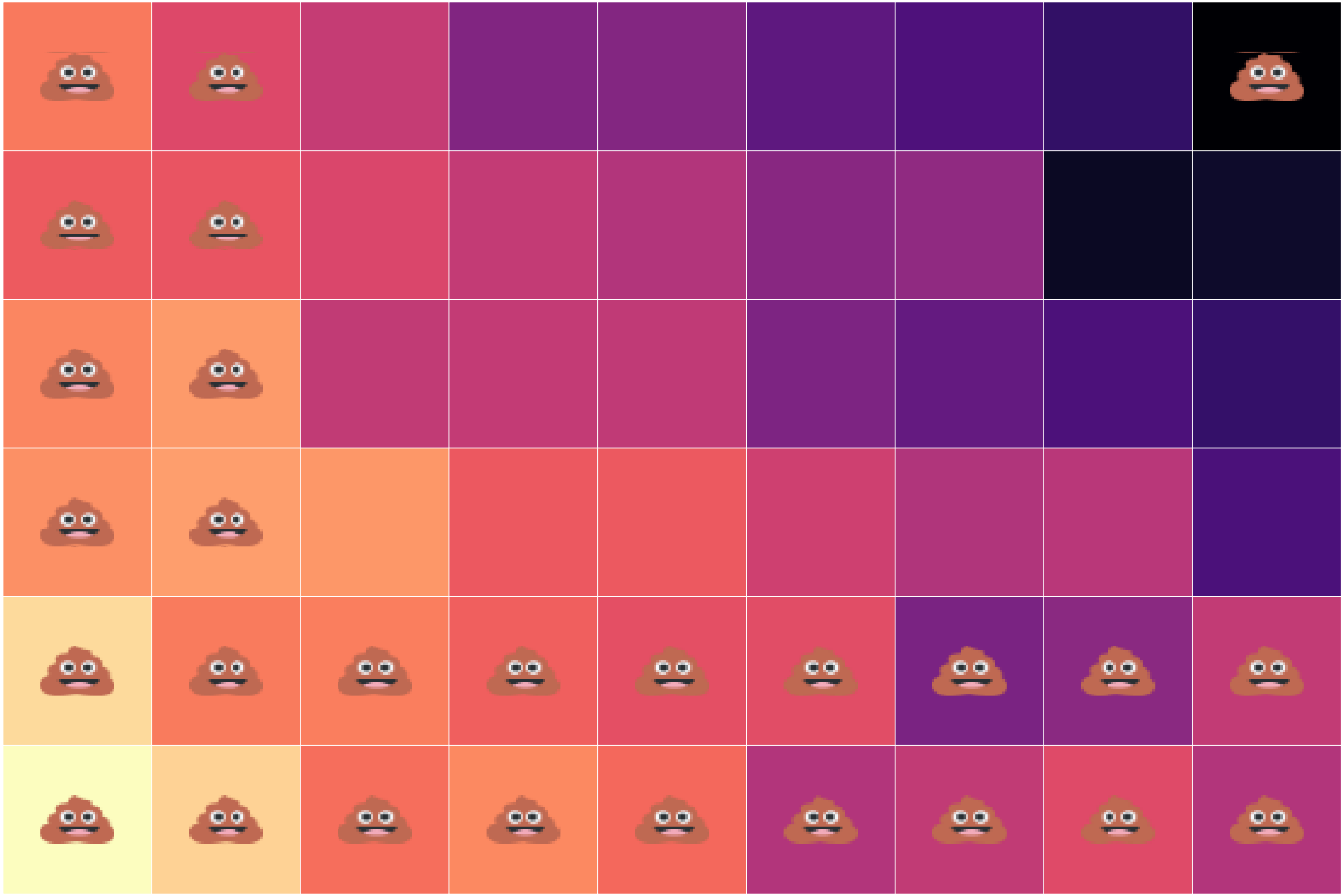
- Require "good" randomization to get *all else equal* (exogeneity).

Note: Your experiment's results are only as good as your randomization.

Unfortunate randomization



Causality

The ideal experiment

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This *ideal experiment* is clearly infeasible[†], but it creates nice notation for causality (the Rubin causal model/Neyman potential outcomes framework).

† Without (1) God-like abilities and multiple universes or (2) a time machine.

The ideal experiment

The *ideal* data for 10 people

```
#>      i trt  y1i  y0i
#> 1    1   1 5.01 2.56
#> 2    2   1 8.85 2.53
#> 3    3   1 6.31 2.67
#> 4    4   1 5.97 2.79
#> 5    5   1 7.61 4.34
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$$\tau_i = y_{1,i} - y_{0,i}$$

for each individual i.

Causality

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#> 1   1   1 5.01 2.56     2.45
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#> 4   4   1 5.97 2.79     3.18
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for each individual i.

The mean of τ_i is the **average treatment effect** (ATE).

Thus, $\overline{\tau} = 3.82$

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The challenge:

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So a dataset that we actually observe for 6 people will look something like

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We can't observe $y_{1,i}$ and $y_{0,i}$.

But, we do observe

- $y_{1,i}$ for i in 1, 2, 3, 4, 5
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Q: How do we "fill in" the `NA`s and estimate $\overline{\tau}$?

Causally estimating the treatment effect

Notation: Let D_i be a binary indicator variable such that

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Time for math! 🎉

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This assumption says that the treatment effect is equal (constant) across all individuals i.

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Note: We defined

$$\tau_i = \tau = y_{1,i} - y_{0,i}$$

which implies

$$y_{1,i} = y_{0,i} + \tau$$

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So our proposed group-difference estimator give us the sum of

1. τ, the **causal, averate treatment effect** that we want
2. **Selection bias:** How much trt. and control groups differ (on average).

Next time: Solving selection bias.

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