Causality

EC 421, Set 10

Edward Rubin
22 May 2019

Prologue

Schedule

Last Time

Autocorrelation and nonstationarity

Today

Causality

Upcoming

Assignment

Causality

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Intro

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2. **Causal estimation:**[†] Estimate the actual data-generating process—learning about the true, population model that explains how y changes when we change x_j—focuses on β_j. Accuracy of $\hat{y}$ is not important.

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For the rest of the term, we will focus on **causally estimating β_j**.

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Causality

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Causality requires us to **hold all else constant** (*ceterus paribus*).

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- What causes some countries to grow and others to decline?
- What caused President Trump's 2016 election?
- How does the number of police officers affect crime?
- What is the effect of better air quality on test scores?
- Do longer prison sentences decrease crime?
- How did cannabis legalization affect mental health/opioid addiction?

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New saying:

> Correlation plus exogeneity is causation.

Let's work through a few examples.

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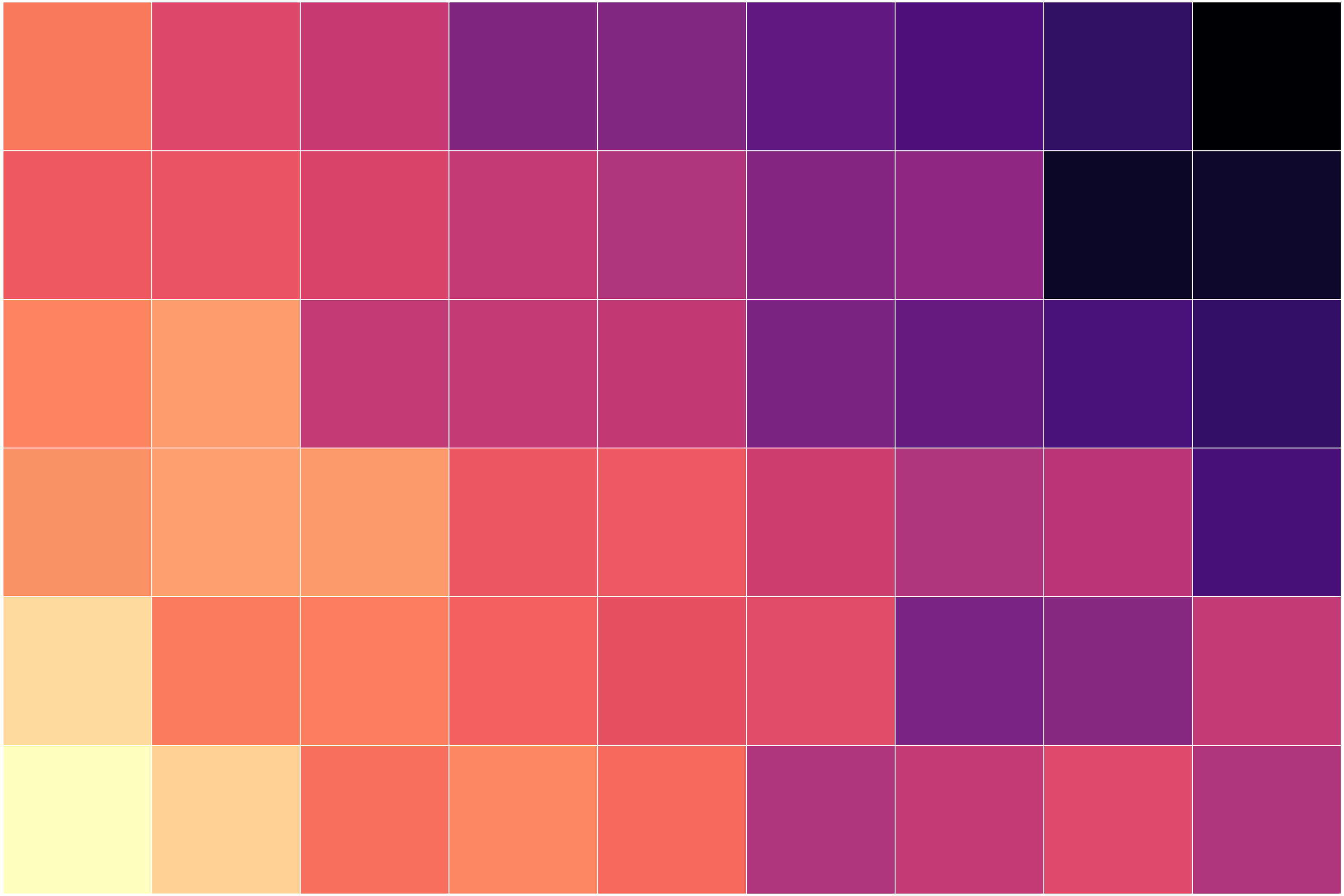
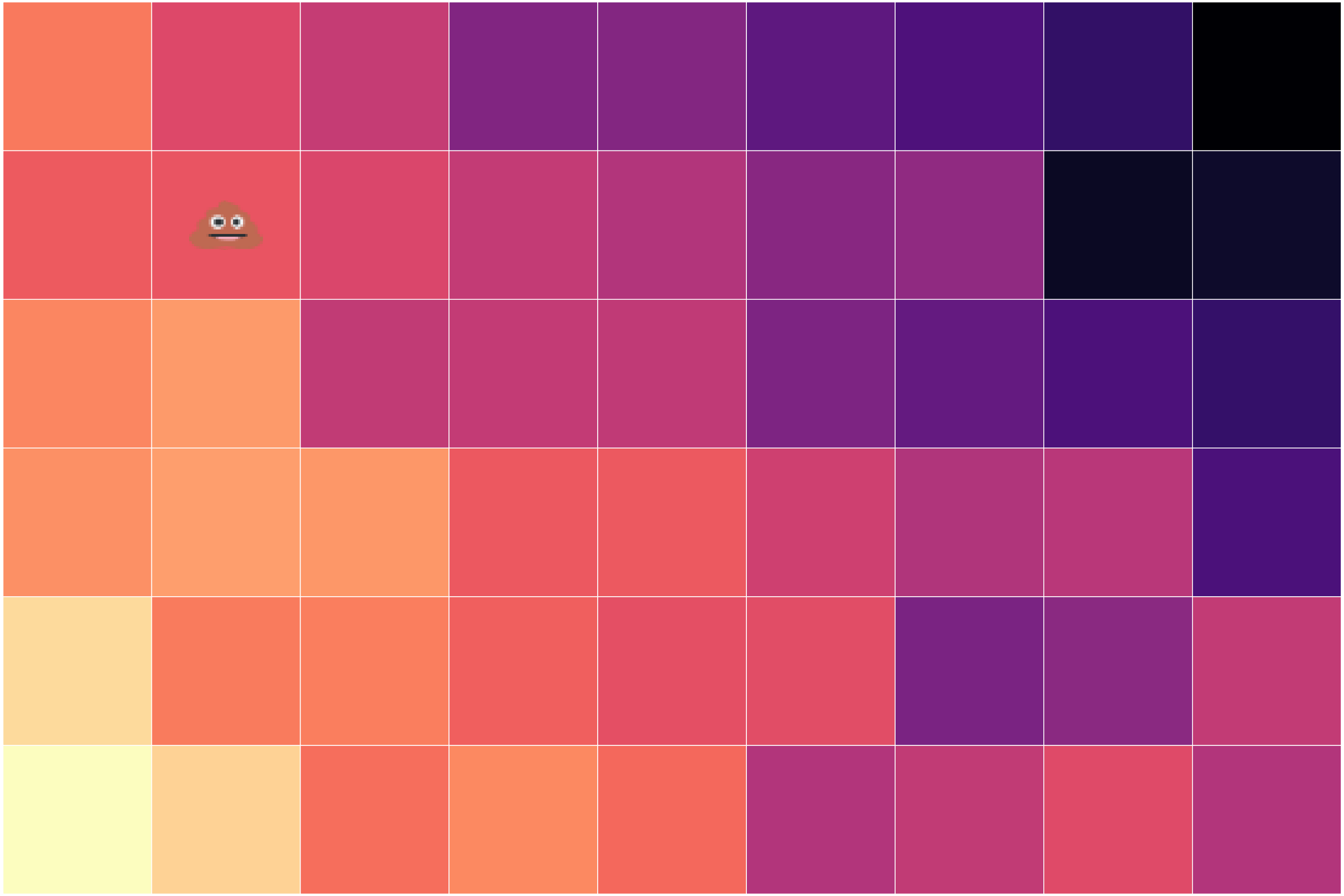
01	02	03	04	05	06	07	08	09
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54

54 equal-sized plots of varying quality



54 equal-sized plots of varying quality plus randomly assigned treatment



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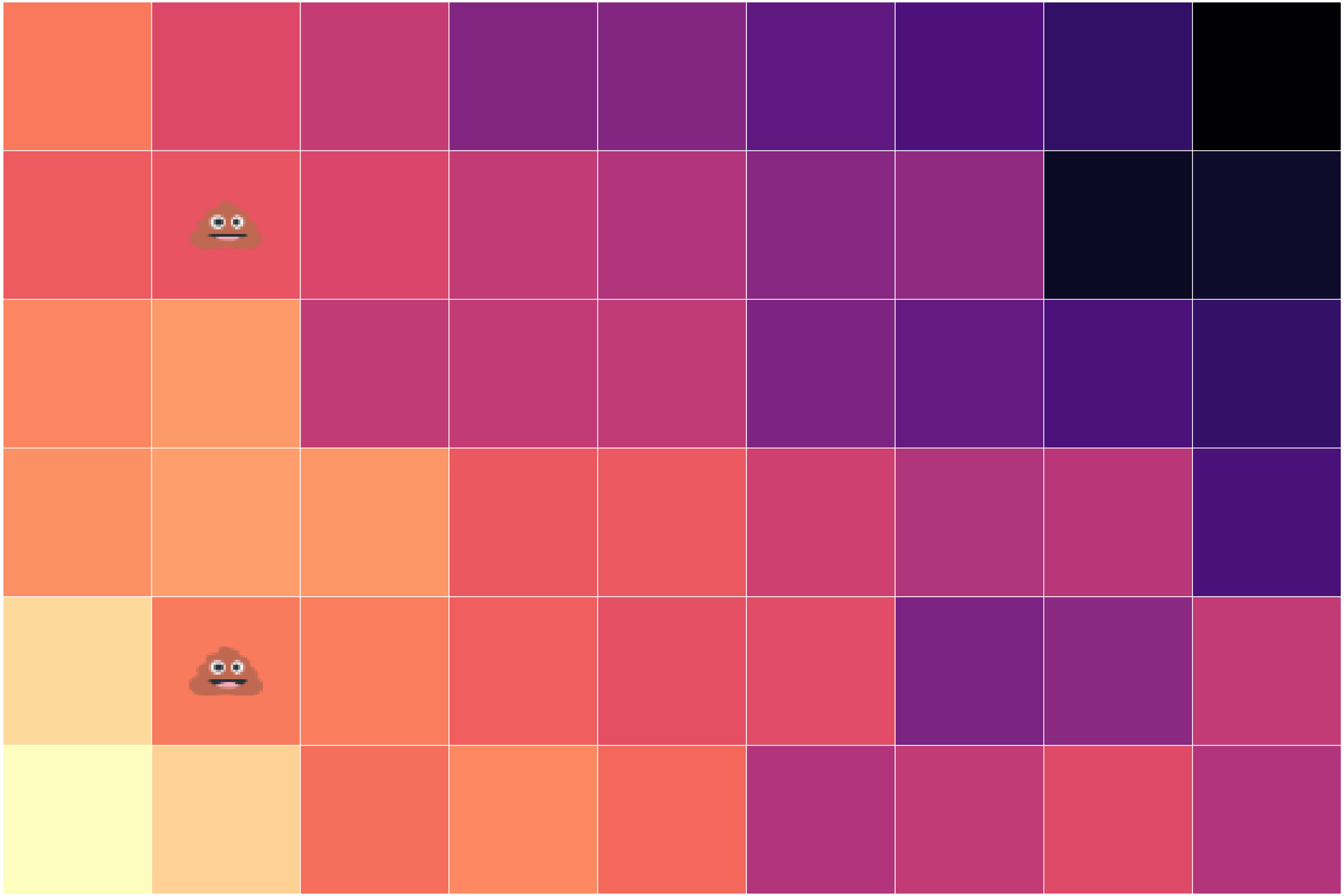

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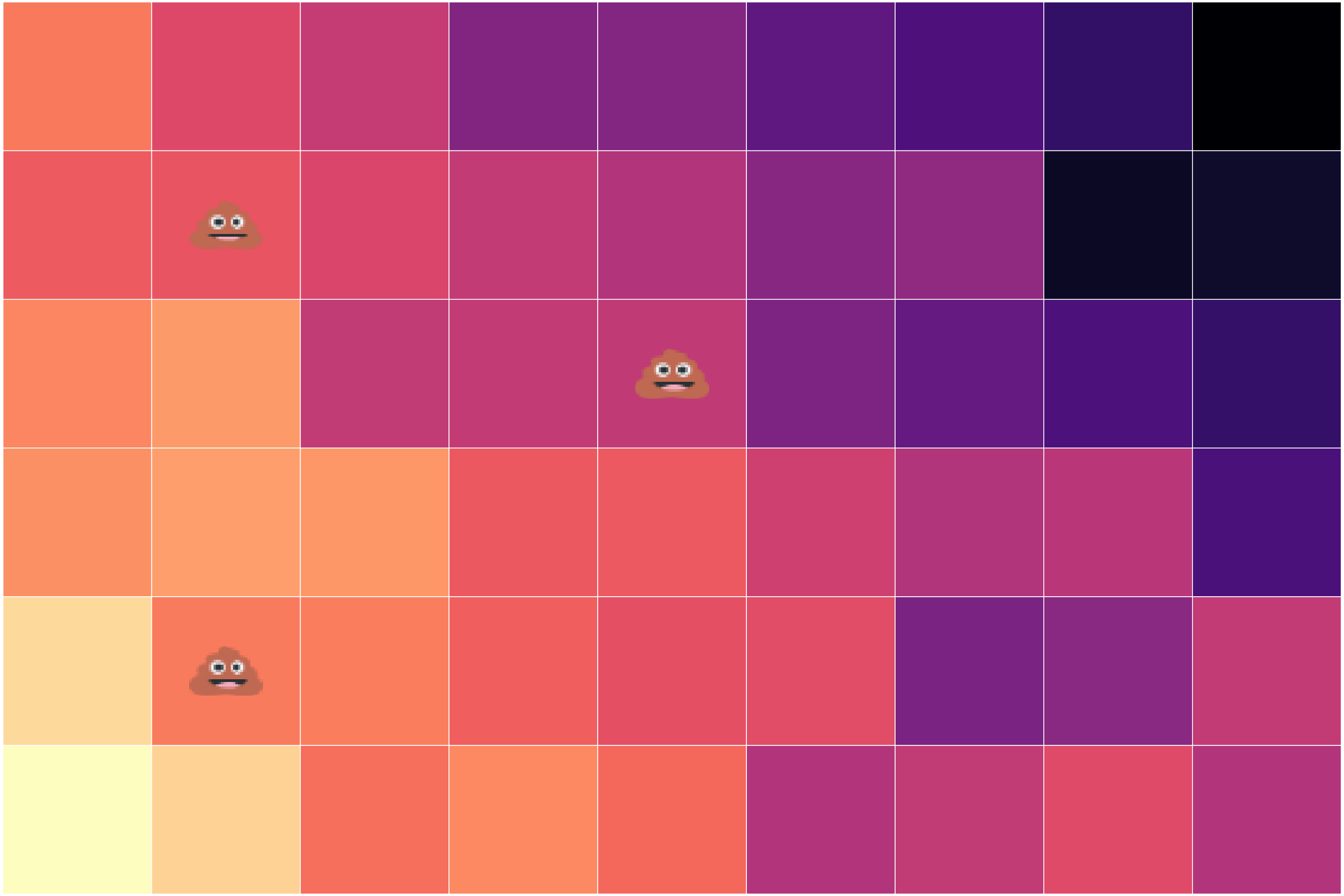

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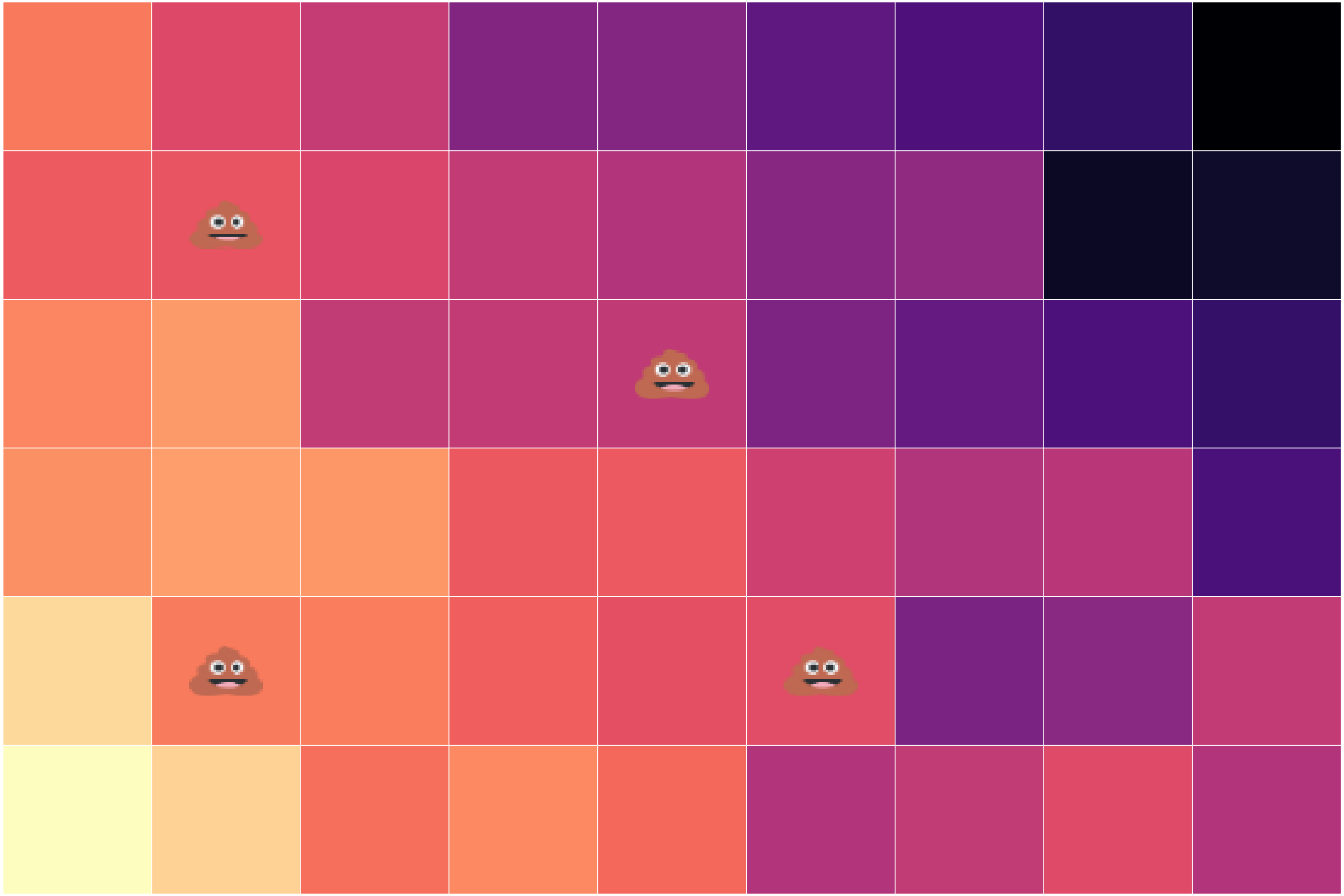

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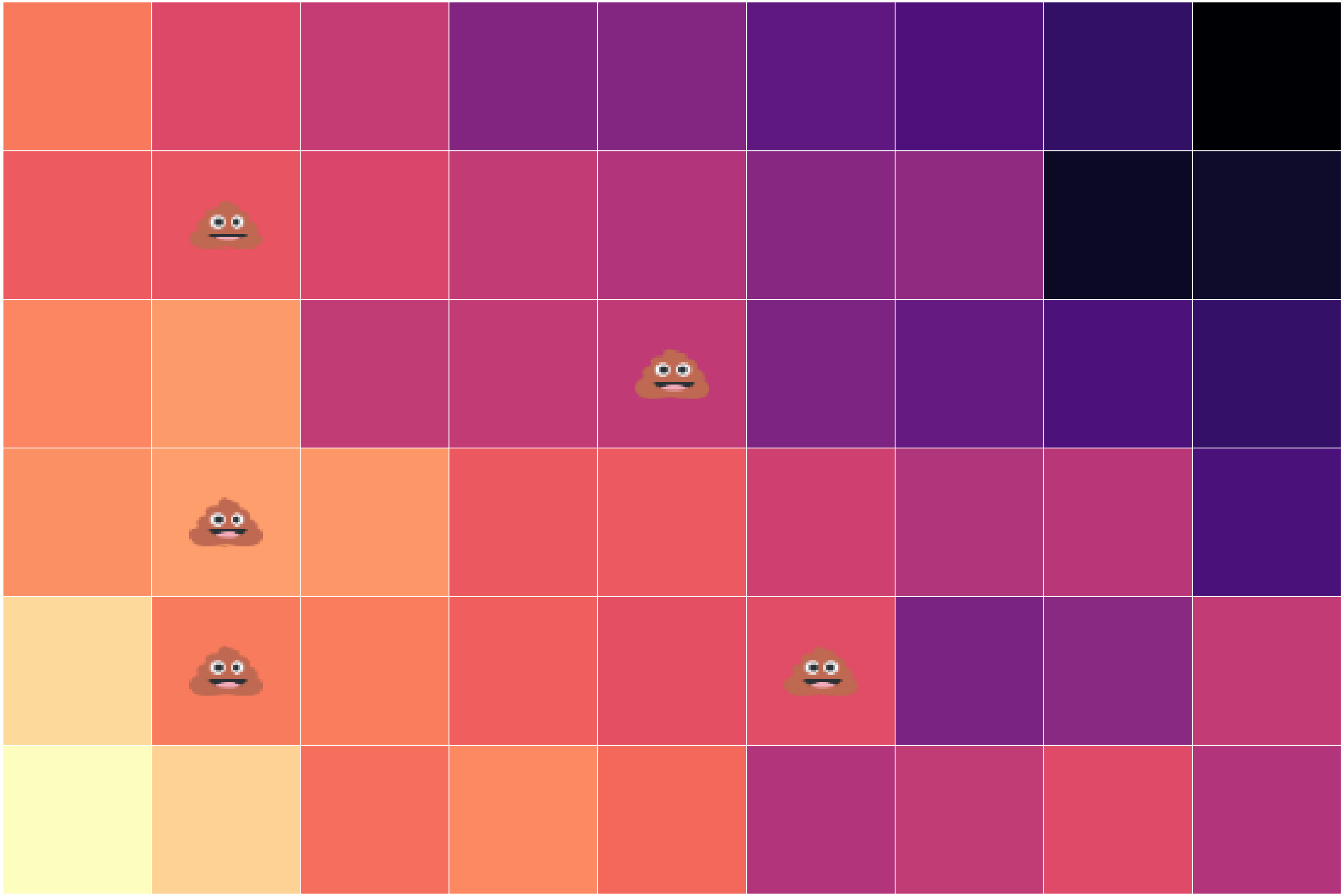

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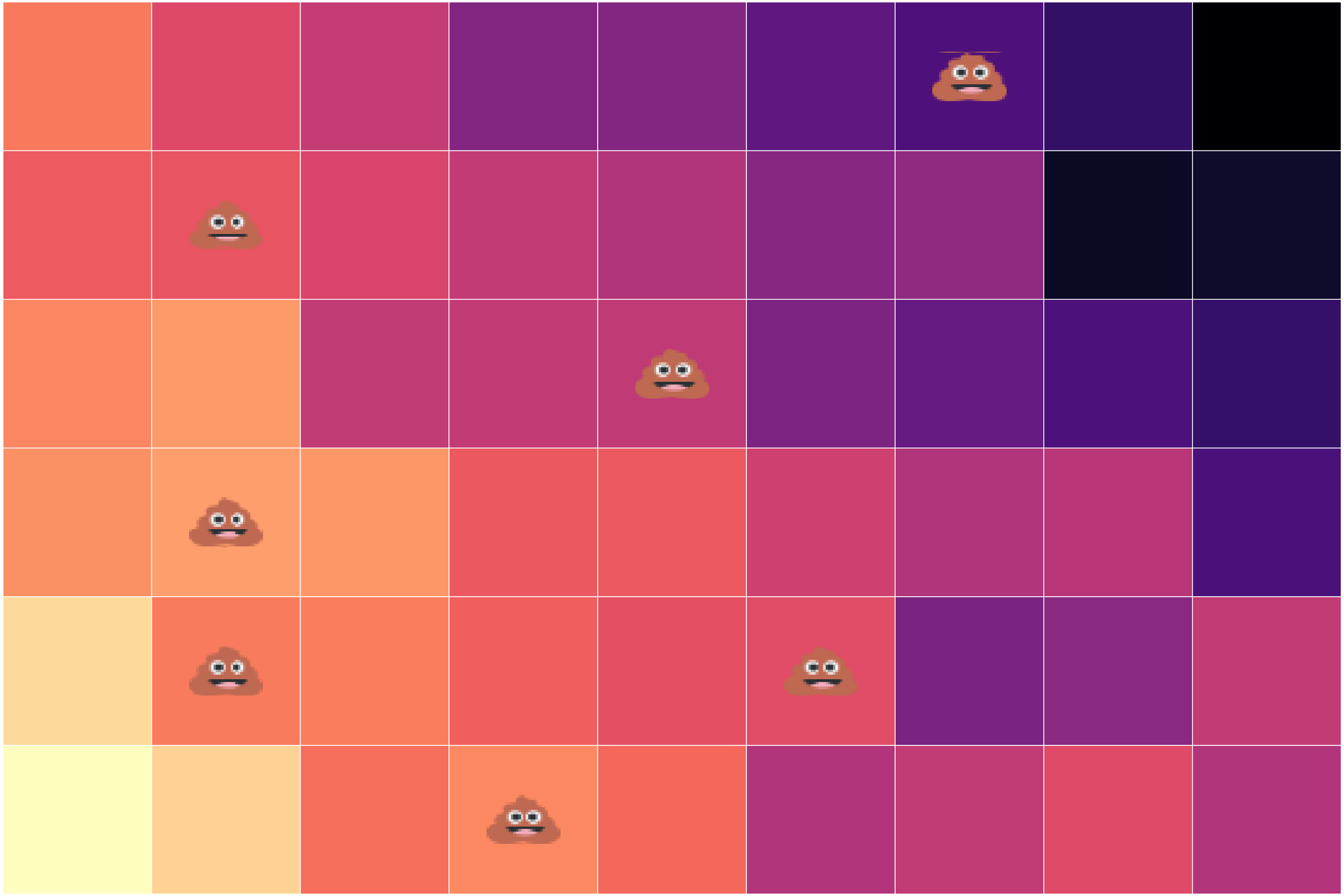

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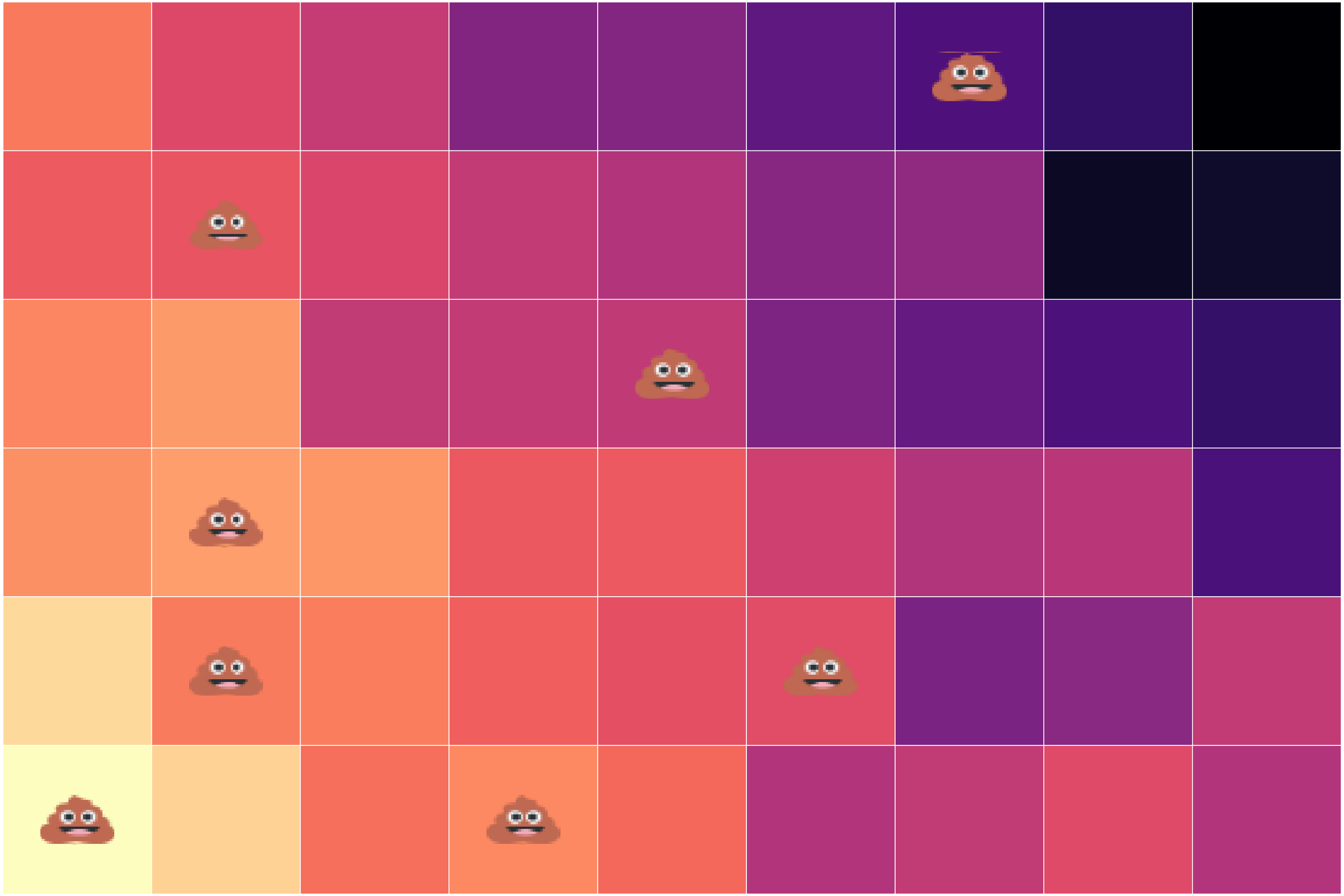

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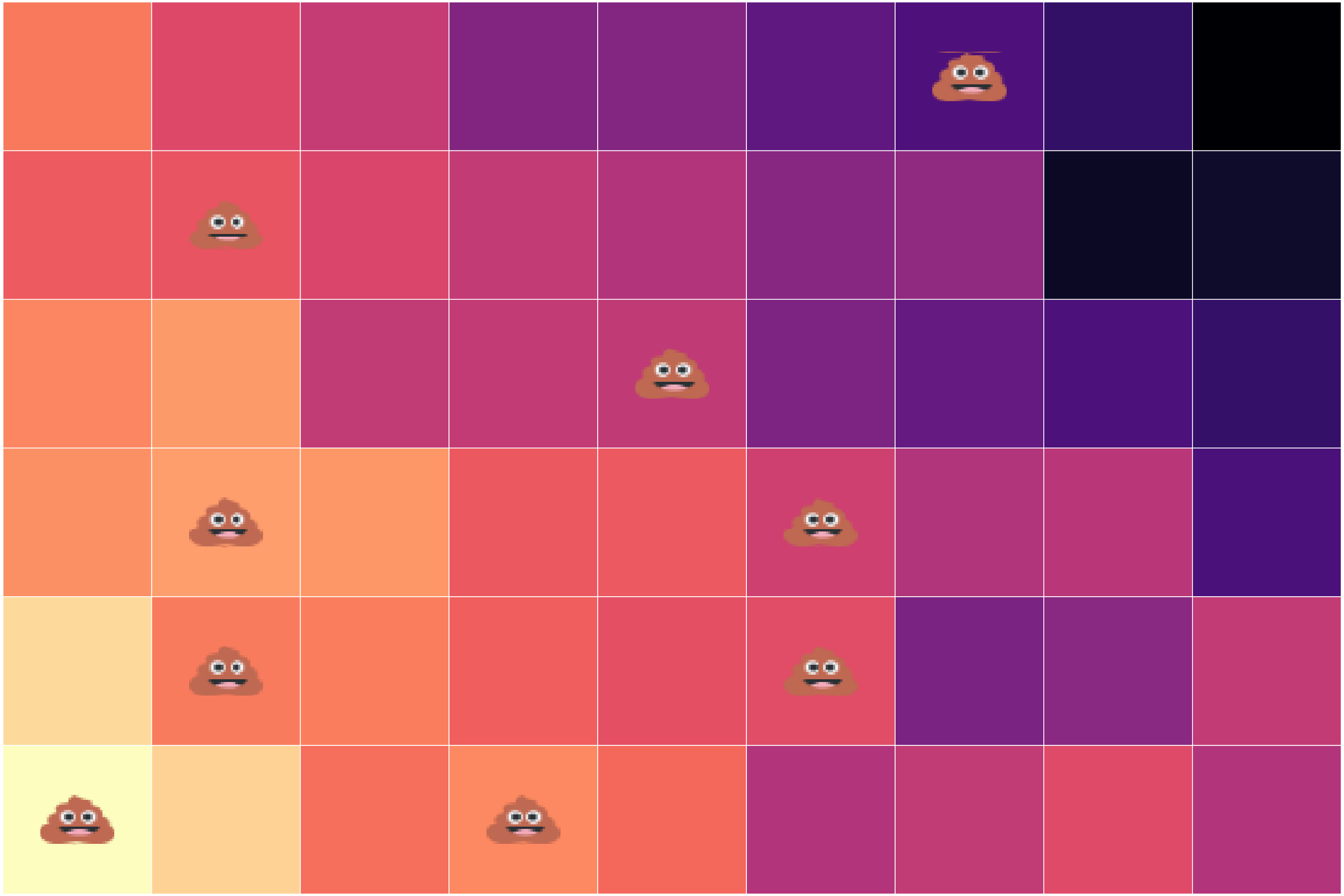

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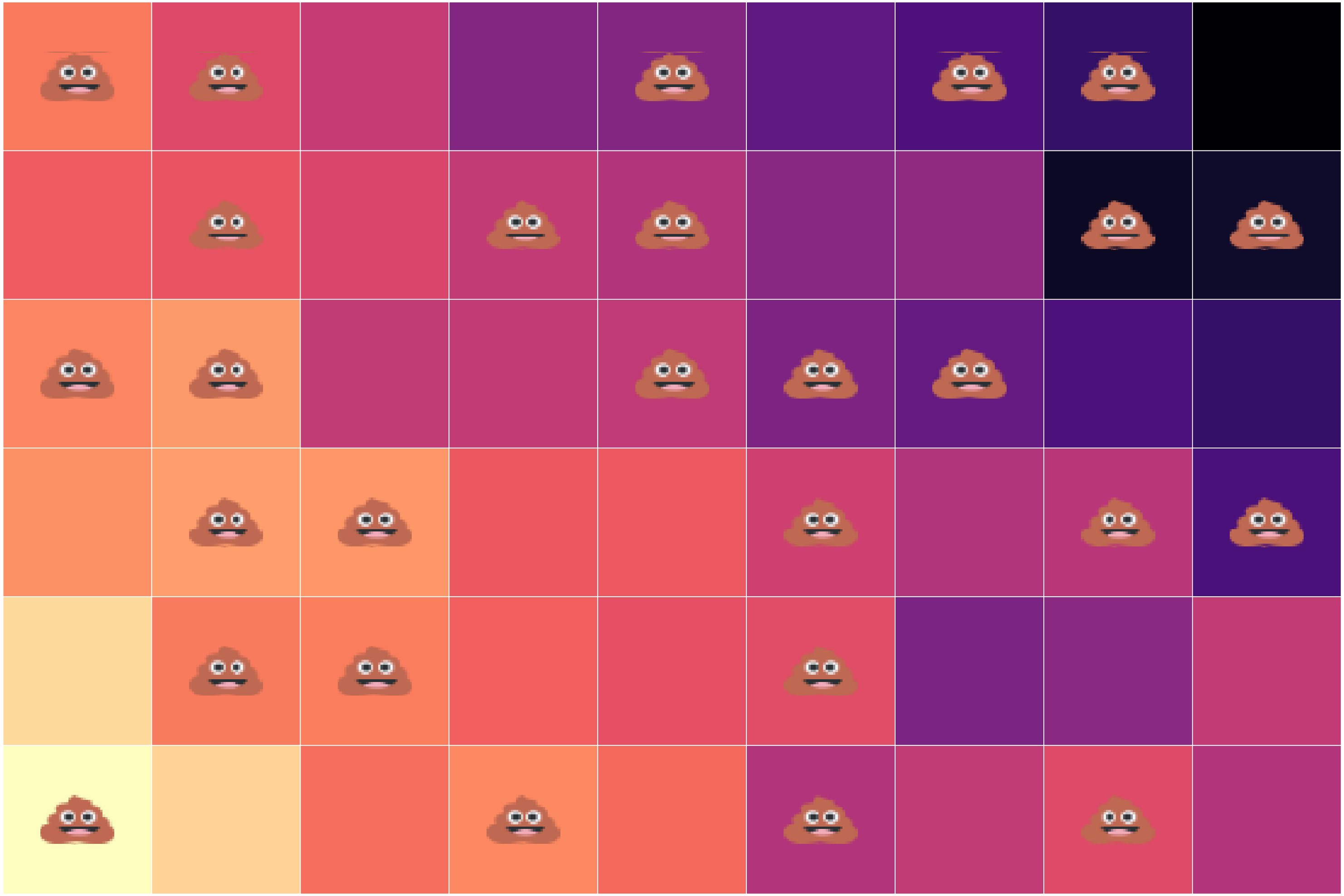

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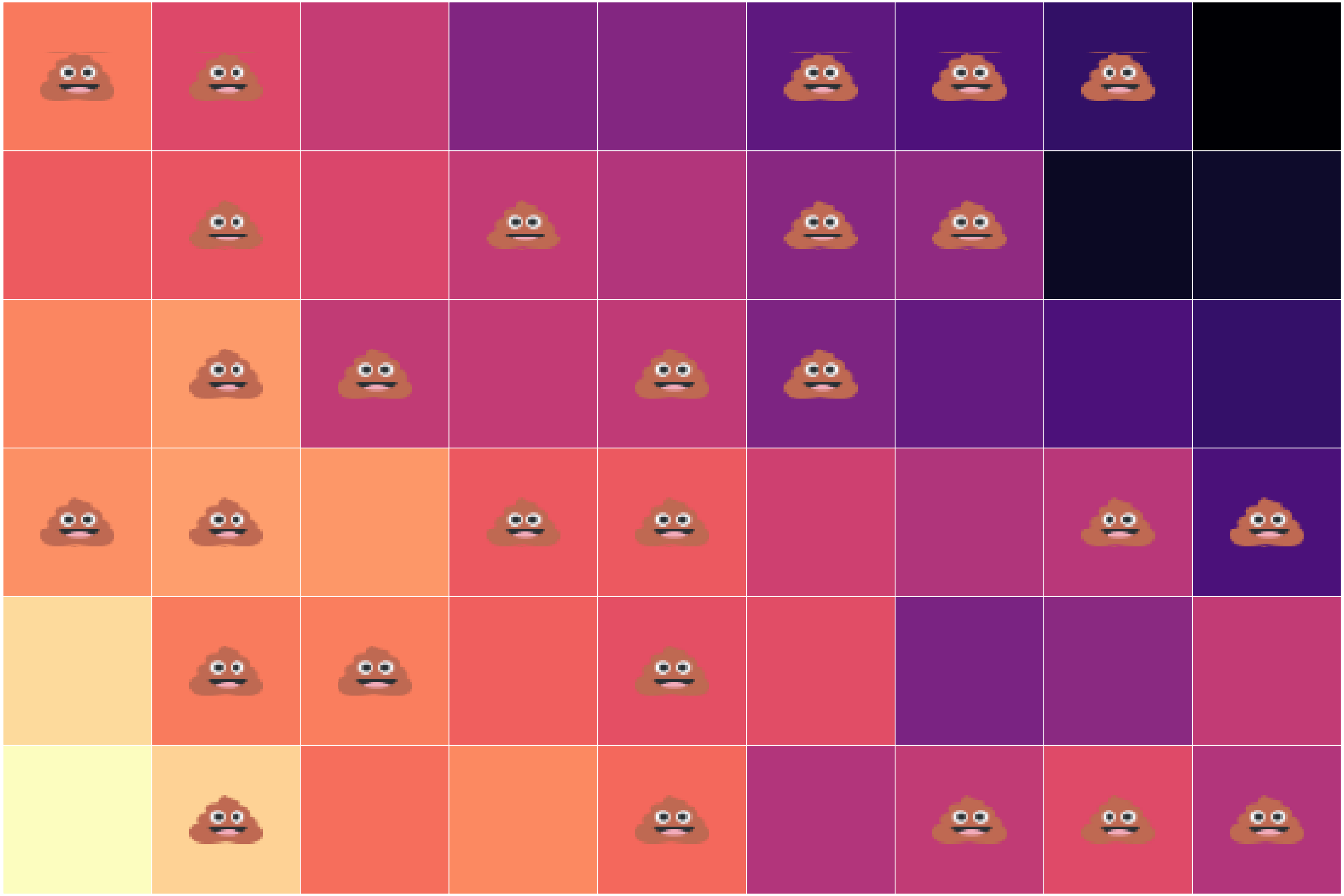

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Example: The causal effect of fertilizer

We can estimate the **causal effect** of fertilizer on crop yield by comparing the average yield in the treatment group (💩) with the control group (no 💩).

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A: On average, **randomly assigning treatment should balance** trt. and control across the other dimensions that affect yield (soil, slope, water).

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Example: Returns to education

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Thought experiment:

- Randomly select an individual.
- Give her an additional year of education.
- How much do her earnings increase?

This change in earnings gives the **causal effect** of education on earnings.

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The point (2) above also illustrates the difficulty in learning about educations while *holding all else constant*.

Many important variables have the same challenge—gender, race, income.

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- Admissions cutoffs
- Lottery enrollment and/or capacity constraints

Real-world experiments

Both examples consider **real experiments** that isolate causal effects.

Characteristics

- Feasible—we can actually (potentially) run the experiment.
- Compare individuals randomized into treatment against individuals randomized into control.
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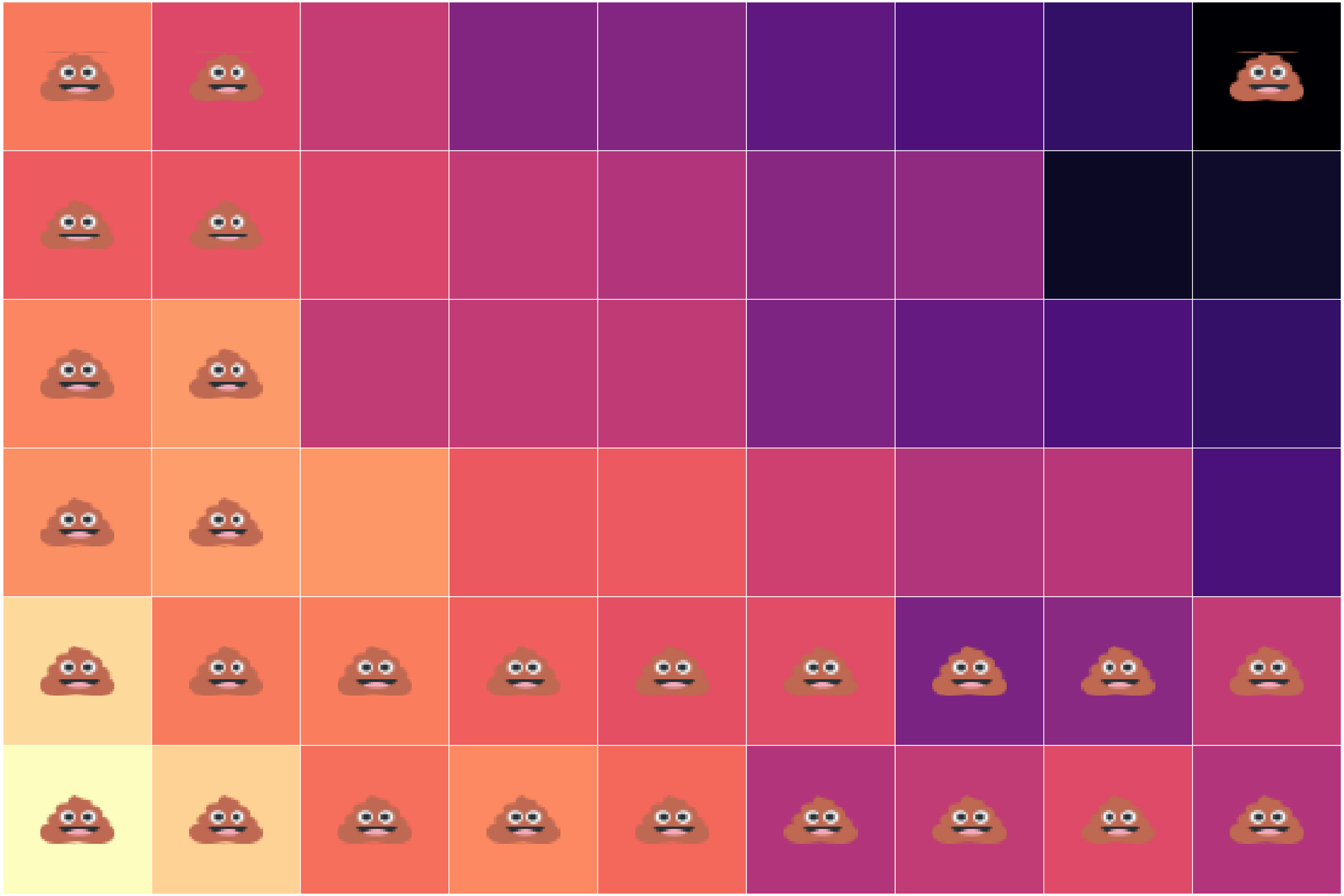
Note: Your experiment's results are only as good as your randomization.

Unfortunate randomization



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This *ideal experiment* is clearly infeasible[†], but it creates nice notation for causality (the Rubin causal model/Neyman potential outcomes framework).

† Without (1) God-like abilities and multiple universes or (2) a time machine.

The ideal experiment

The *ideal* data for 10 people

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#>     i trt  y1i  y0i
#> 1   1   1 5.01 2.56
#> 2   2   1 8.85 2.53
#> 3   3   1 6.31 2.67
#> 4   4   1 5.97 2.79
#> 5   5   1 7.61 4.34
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The mean of τ_i is the **average treatment effect** (ATE).

Thus, $\bar{\tau} = 3.82$

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We can't observe $y_{1,i}$ and $y_{0,i}$.

But, we do observe

- $y_{1,i}$ for i in 1, 2, 3, 4, 5
- $y_{0,j}$ for j in 6, 7, 8, 9, 10

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```
#>      i trt  y1i  y0i
#> 1    1   1 5.01   NA
#> 2    2   1 8.85   NA
#> 3    3   1 6.31   NA
#> 4    4   1 5.97   NA
#> 5    5   1 7.61   NA
#> 6    6   0   NA 4.15
#> 7    7   0   NA 0.56
#> 8    8   0   NA 3.52
#> 9    9   0   NA 4.49
#> 10  10   0   NA 1.40
```

We can't observe $y_{1,i}$ and $y_{0,i}$.

But, we do observe

- $y_{1,i}$ for i in 1, 2, 3, 4, 5
- $y_{0,j}$ for j in 6, 7, 8, 9, 10

Q: How do we "fill in" the `NA`s and estimate $\overline{\tau}$?

Causally estimating the treatment effect

Notation: Let D_i be a binary indicator variable such that

- $D_i = 1$ if individual i is treated.
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Causality

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Time for math! 🎉

Causality

Causally estimating the treatment effect

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Note: We defined

$$\tau_i = \tau = y_{1,i} - y_{0,i}$$

which implies

$$y_{1,i} = y_{0,i} + \tau$$

$Q_{3.0}$: Is $Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0)$ a *good* estimator for τ?

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Difference in groups' means

$$
\begin{aligned}
&= Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0) \\
&= Avg(y_{1,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0) \\
&= Avg(\tau + y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0) \\
&= \tau + Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0) \\
&= \text{Average causal effect} + \text{Selection bias}
\end{aligned}
$$

So our proposed group-difference estimator give us the sum of

1. τ, the **causal, average treatment effect** that we want
2. **Selection bias:** How much trt. and control groups differ (on average).

Next time: Solving selection bias.

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