Non-Stationary Time Series

EC 421, Set 9

Edward Rubin
22 May 2019

Prologue

Schedule

Last Time

Autocorrelation

Today

- Brief introduction to nonstationarity
- Then: Causality

Upcoming

- **Assignment** this afternoon.

Nonstationarity

Nonstationarity

Intro

Let's go back to our assumption of **weak dependence/persistence**

> 1. **Weakly persistent outcomes**—essentially, x_{t+k} in the distant period $t+k$ weakly correlates with x_t (when k is "big").

Nonstationarity

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We'll define this *good behavior* as **stationarity**.

Stationarity

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3. The **covariance** between x_t and x_{t-k} depends only on k—not on t, *i.e.*,

$$\mathrm{Cov}(x_t,\, x_{t-k}) = \mathrm{Cov}(x_s,\, x_{s-k}) \text{ for all } t \text{ and } s$$

Nonstationarity

Random walks

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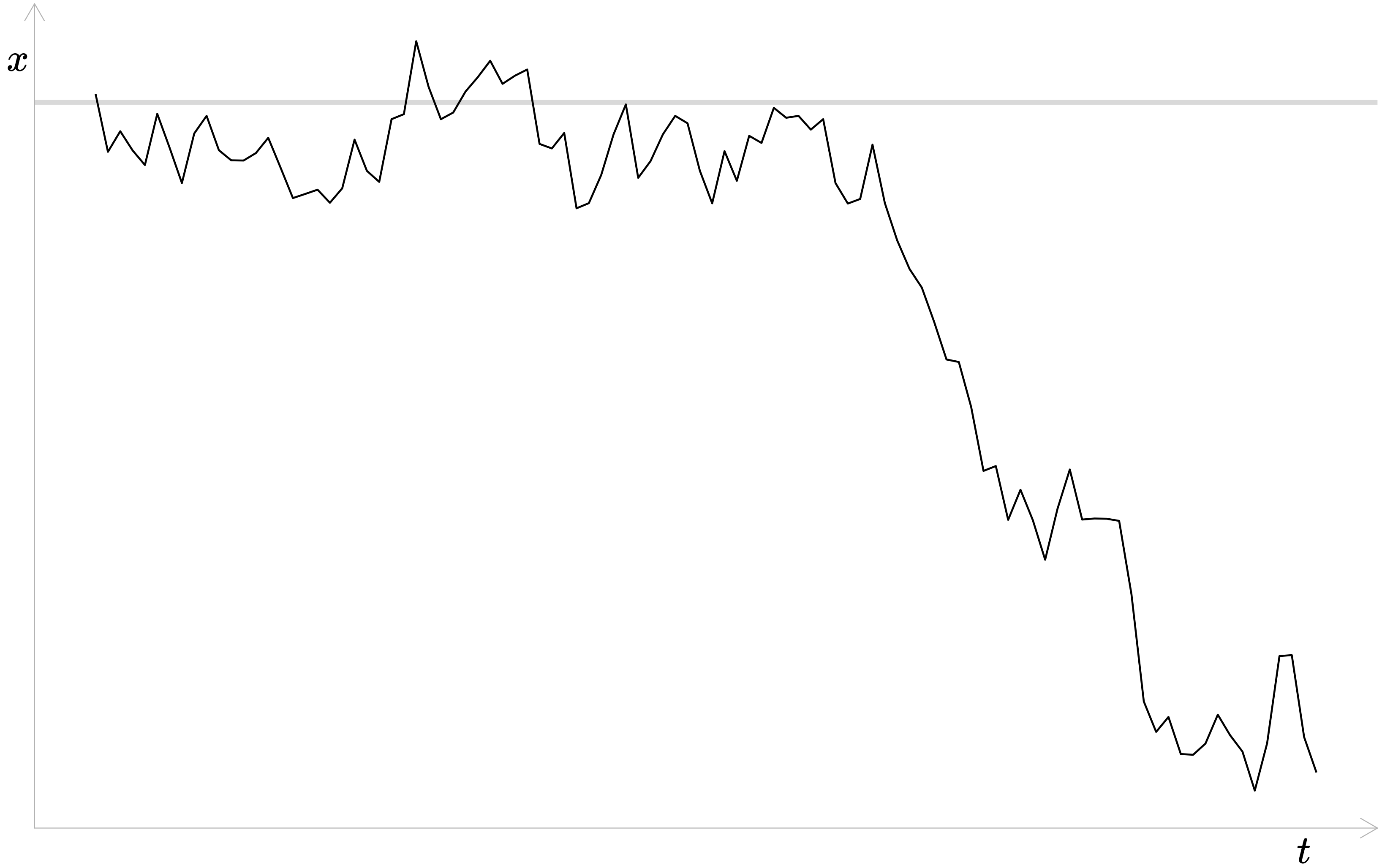
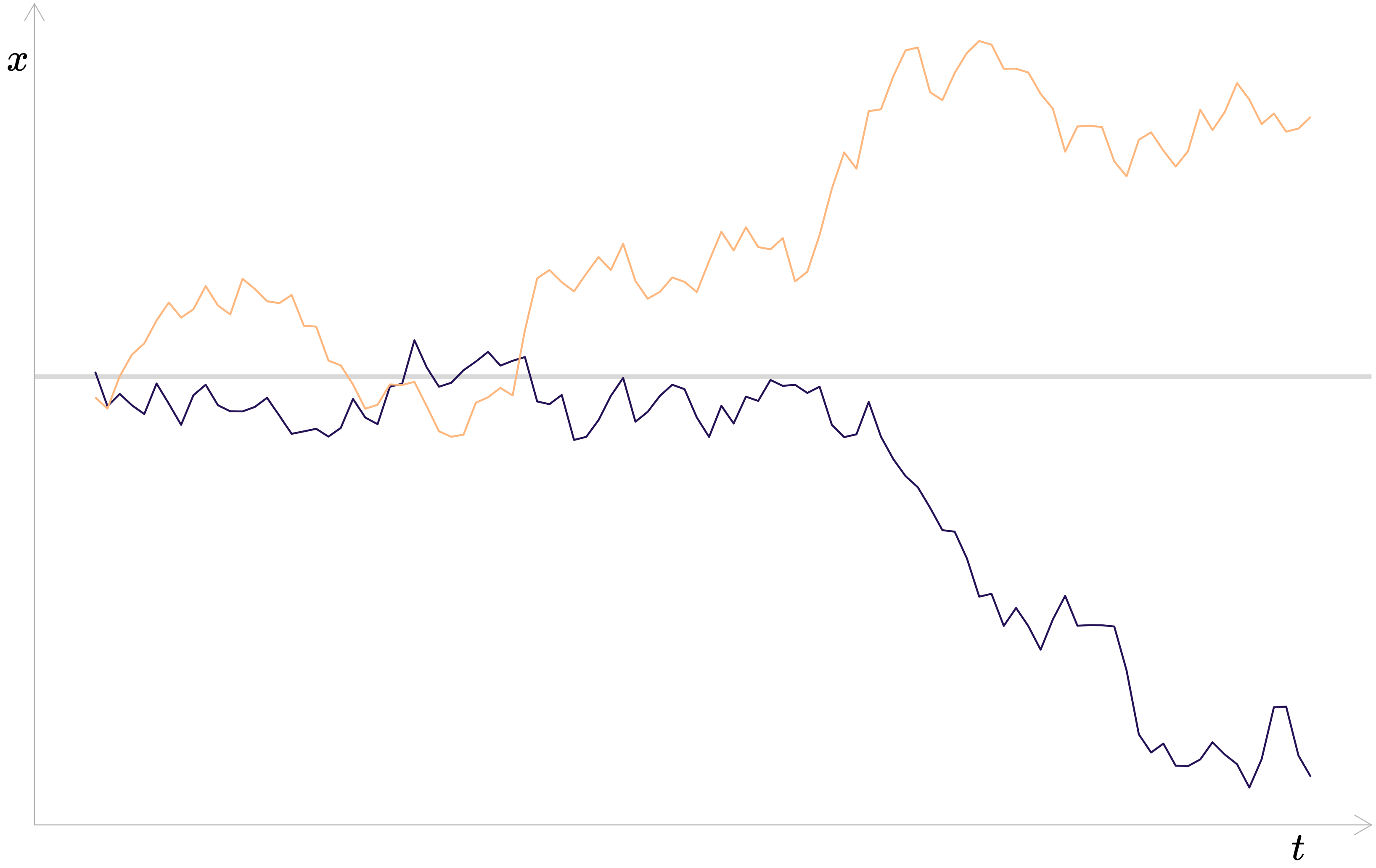
$$\begin{aligned}
\mathbf{Var}(x_t) &= \mathbf{Var}(x_{t-1} + \varepsilon_t) \\
&= \mathbf{Var}(x_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\
&= \mathbf{Var}(x_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\
&\dots \\
&= \mathbf{Var}(x_0 + \varepsilon_1 + \cdots + \varepsilon_{t_2} + \varepsilon_{t-1} + \varepsilon_t) \\
&= \sigma_\varepsilon^2 + \cdots + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \\
&= t\sigma_\varepsilon^2
\end{aligned}$$

Q: What's the big deal with this violation?

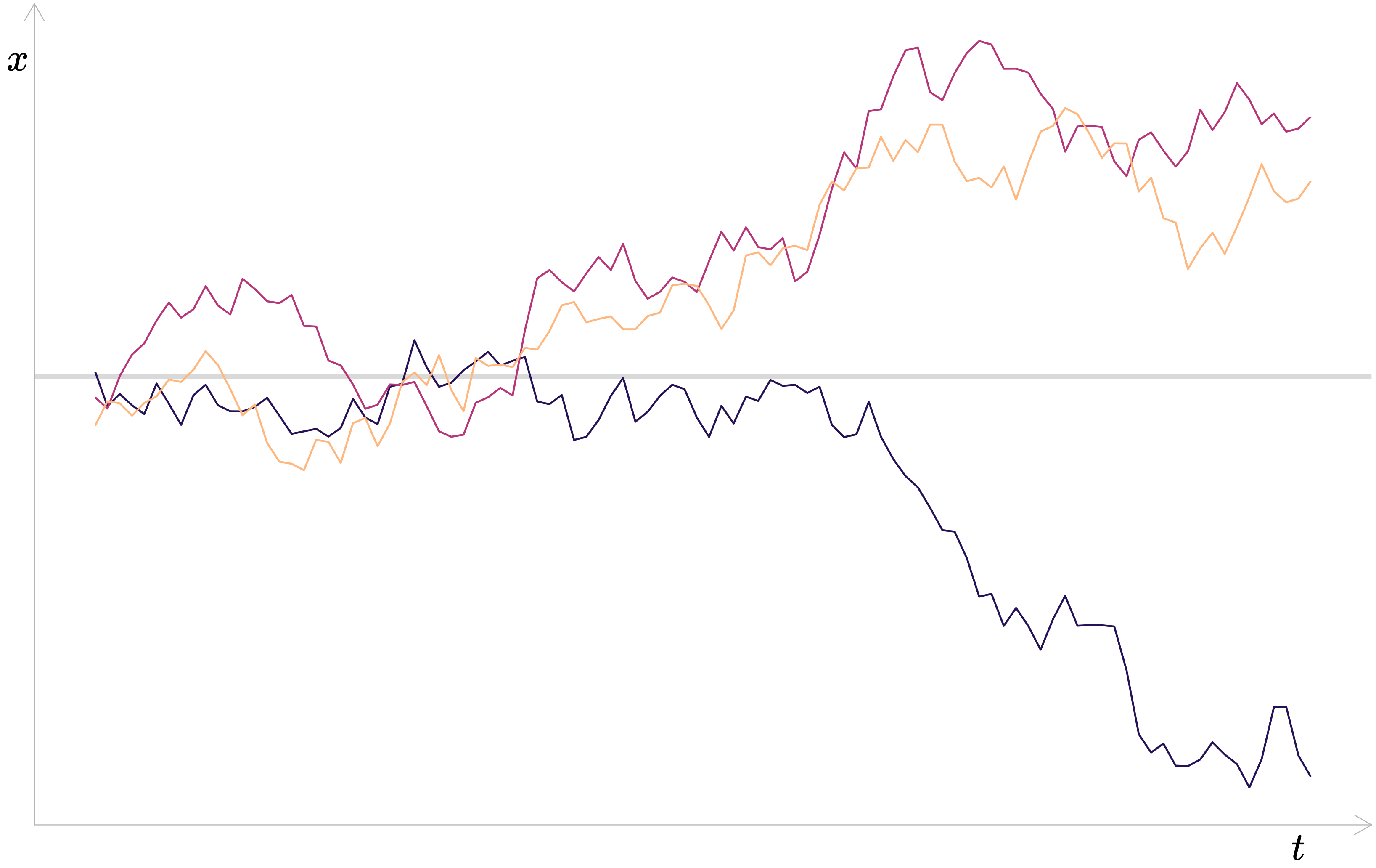
One 100-period random walk



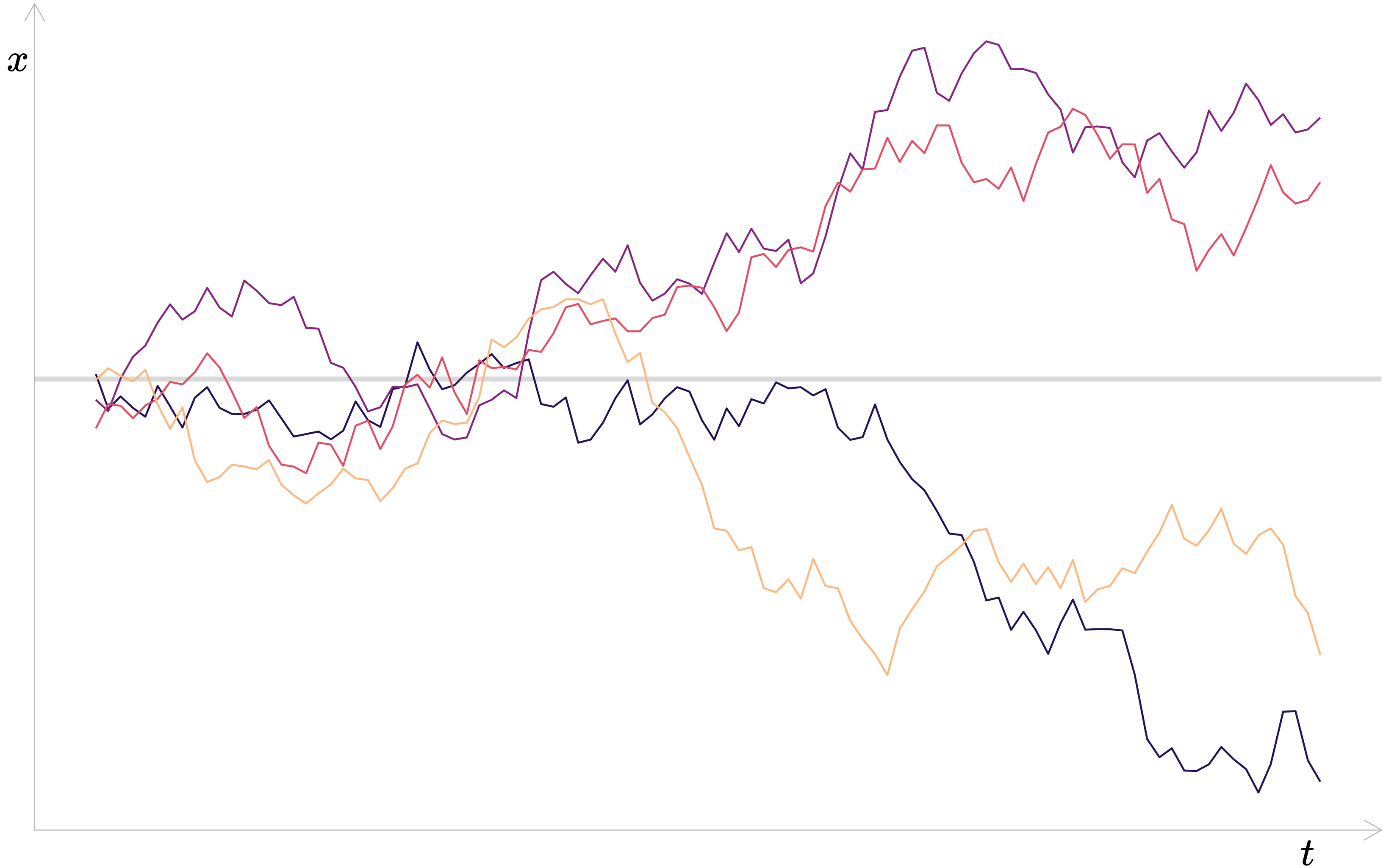
Two 100-period random walks



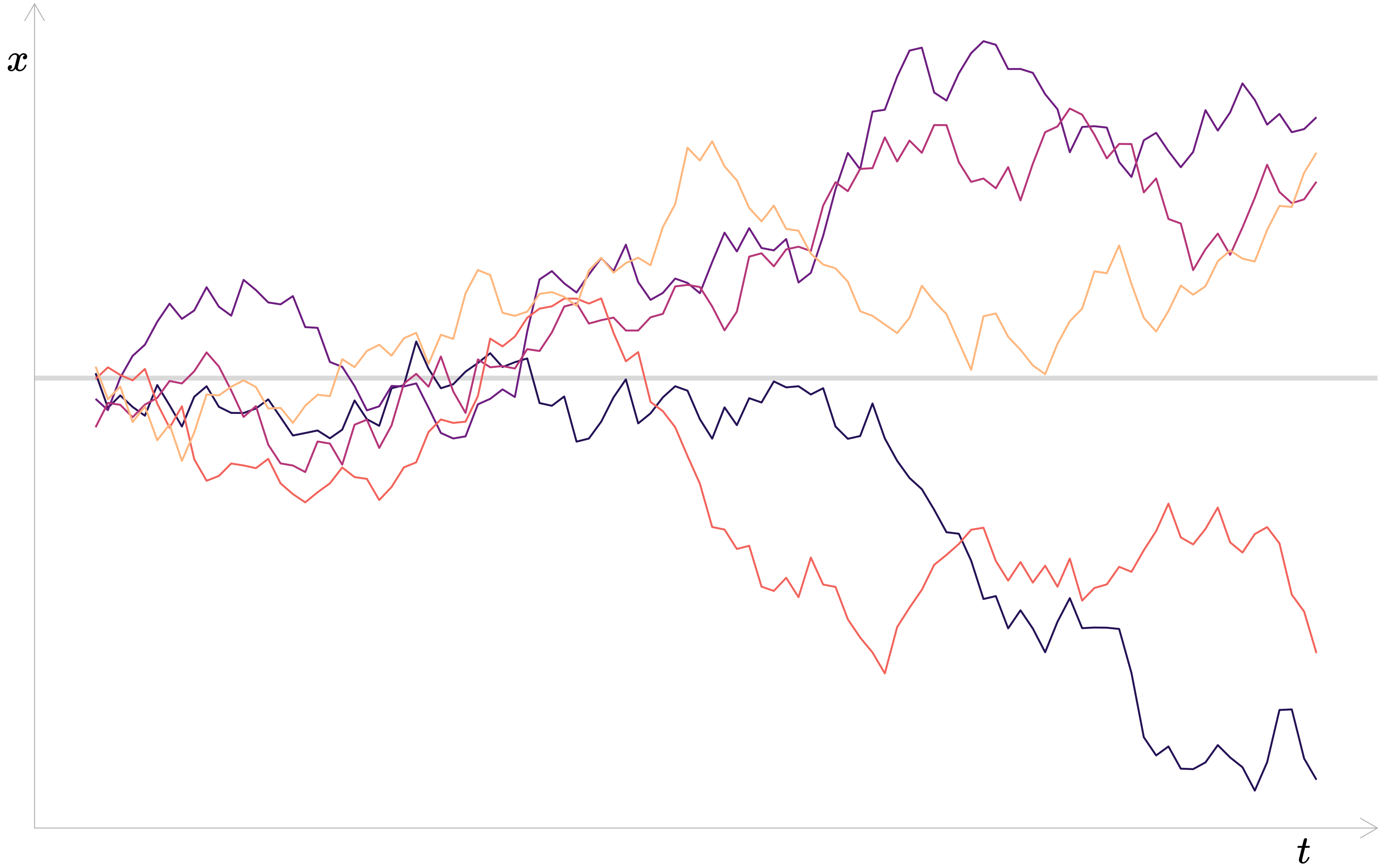
Three 100-period random walks



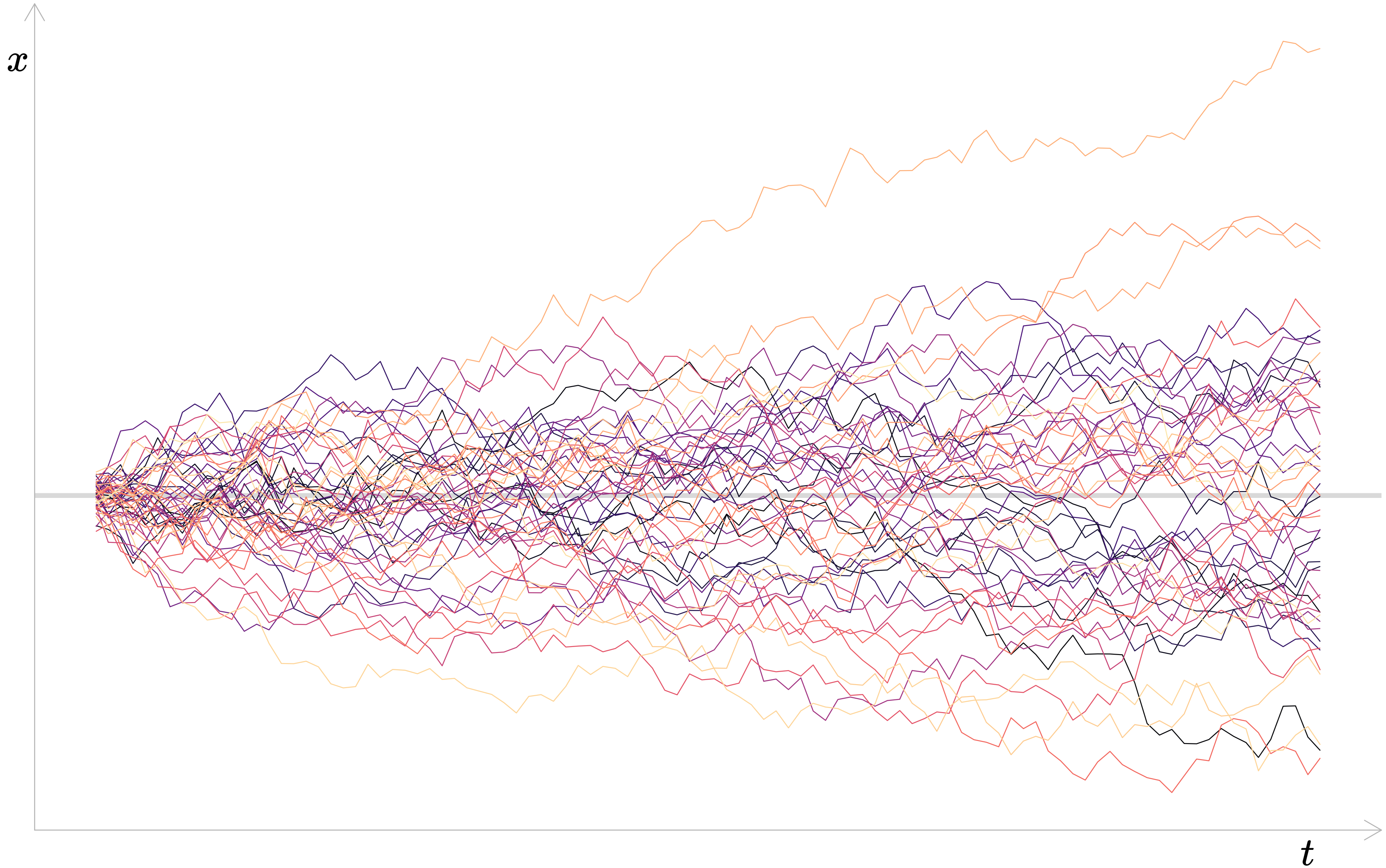
Four 100-period random walks



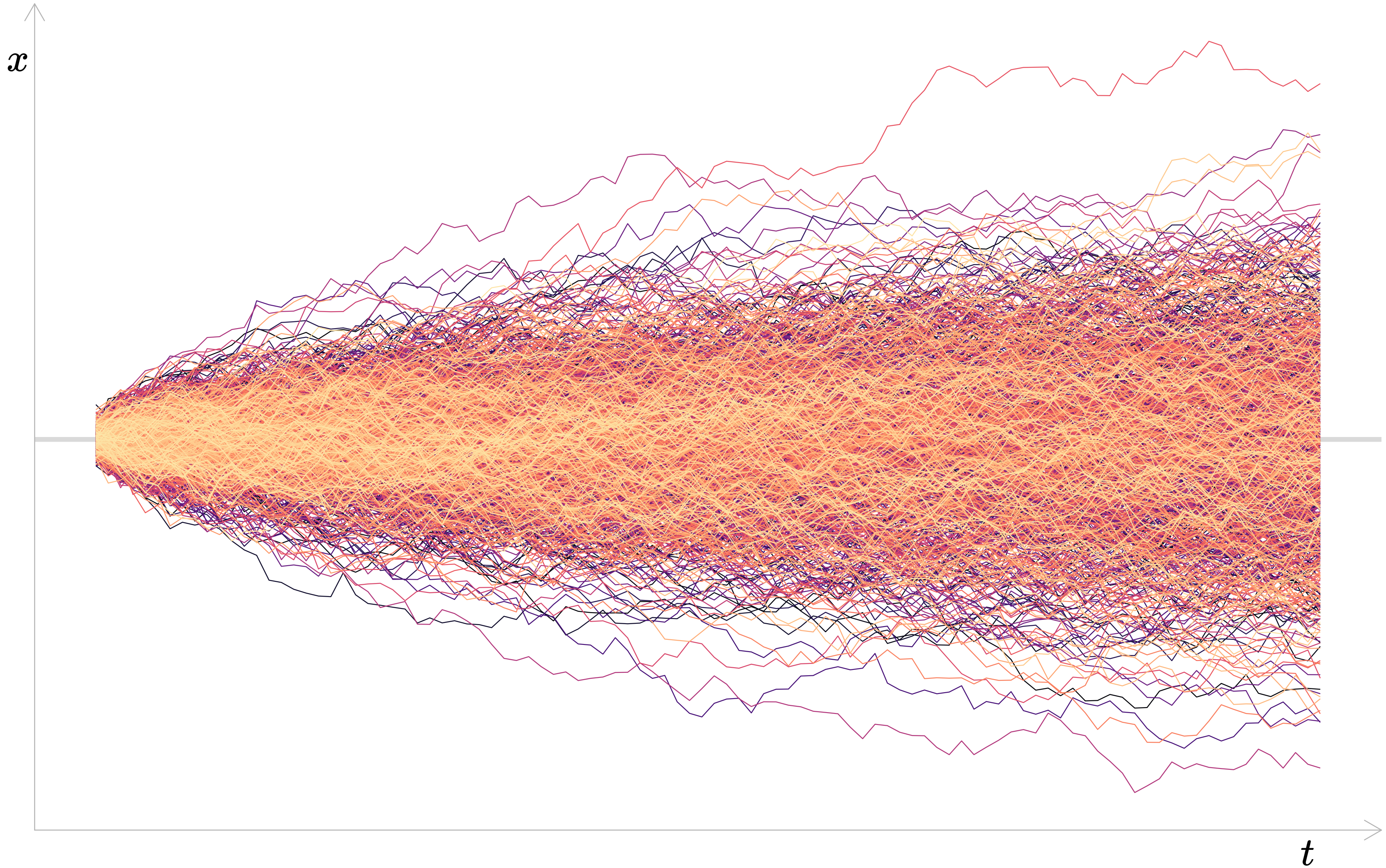
Five 100-period random walks



Fifty 100-period random walks



1,000 100-period random walks



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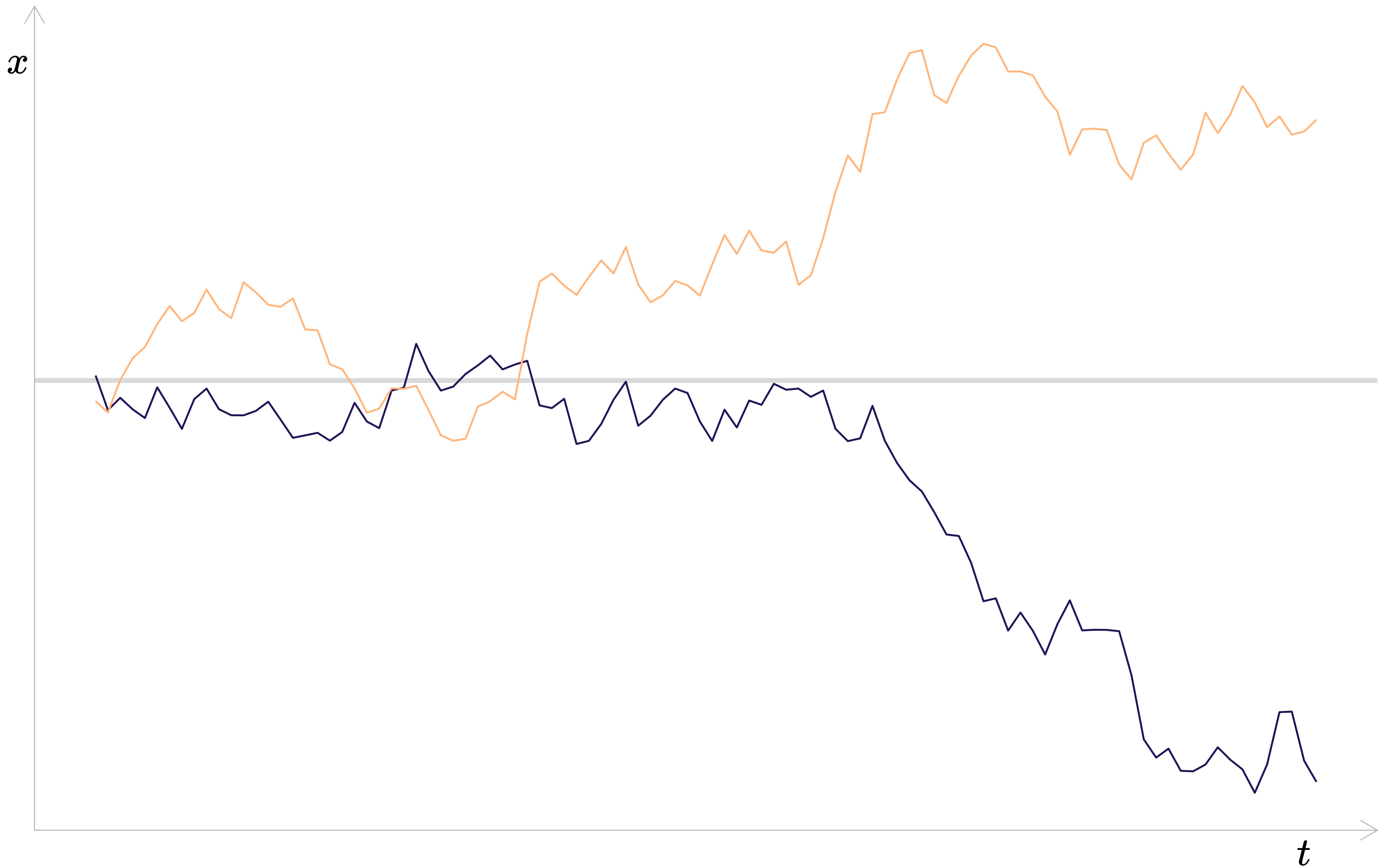
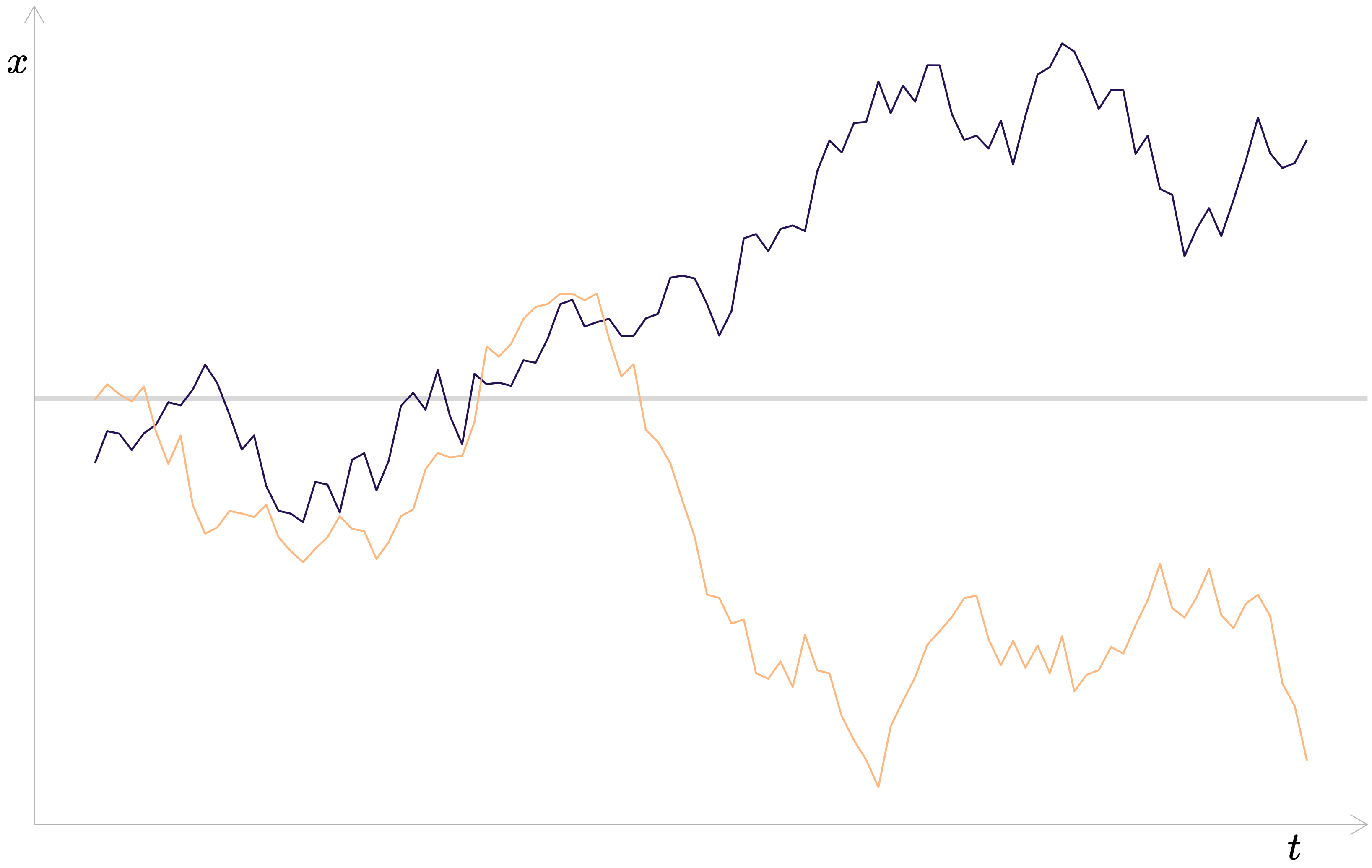
Back in 1974, Granger and Newbold showed that when they **generated random walks** and **regressed the random walks on each other**, **77/100 regressions were statistically significant** at the 5% level (should have been approximately 5/100).

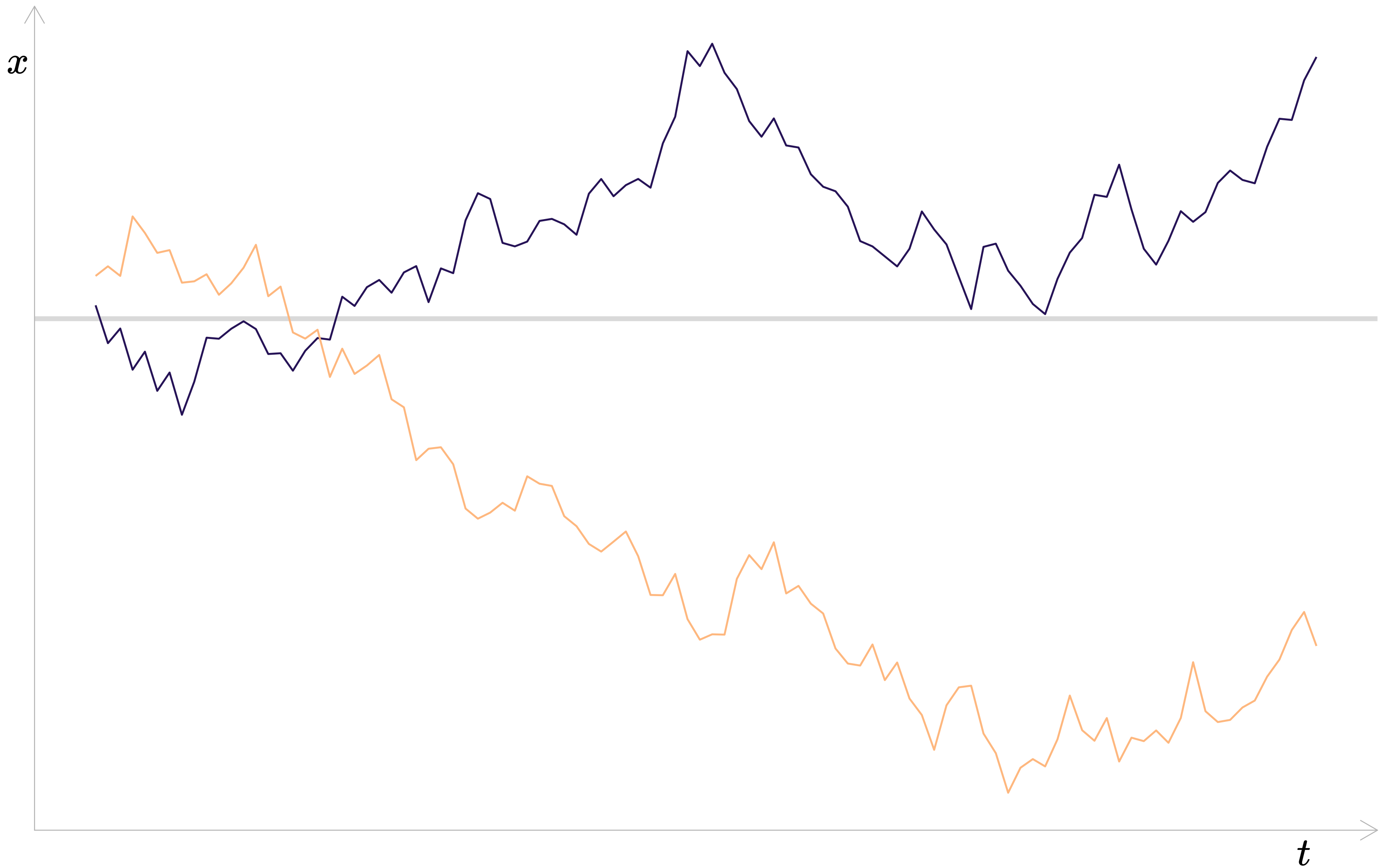
Granger and Newbold simulation example: t statistic ≈ -10.58



Granger and Newbold simulation example: t statistic ≈ -8.92



Granger and Newbold simulation example: t statistic ≈ -7.23



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Deterministic trend: $u_t = \alpha_0 + \beta_1 t + \varepsilon_t$

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A potential solution

Some processes are **difference stationary**, which means we can get back to our stationarity (good behavior) requirement by taking the difference between u_t and u_{t-1}.

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So if we have good reason to believe that our disturbances follow a random walk, we can use OLS on the differences, *i.e.*,

$$
\begin{aligned}
y_t &= \beta_0 + \beta_1 x_t + u_t \\
y_{t-1} &= \beta_0 + \beta_1 x_{t-1} + u_{t-1} \\
y_t - y_{t-1} &= \beta_1 \left(x_t - x_{t-1}\right) + \left(u_t - u_{t-1}\right) \\
\Delta y_t &= \beta_1 \Delta x_t + \Delta u_t
\end{aligned}
$$

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Testing

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Dickey-Fuller tests compare

H$_0$: $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$ with $|\beta_1| < 1$ (**stationarity**)
H$_a$: $y_t = y_{t-1} + \varepsilon_t$ (**random walk**)

Testing

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Dickey-Fuller tests compare

$\mathrm{H_o}$: $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$ with $|\beta_1| < 1$ (**stationarity**)
$\mathrm{H_a}$: $y_t = y_{t-1} + \varepsilon_t$ (**random walk**)

using a t test that $|\beta_1| < 1$.†

† People often just test $\beta_1 < 1$.

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