EC 320 Problem Set 4

Winter 2022

1. (Textbook Question 6.6) 10 points

In a Monte Carlo experiment, a variable Y is generated as a linear function of two variables X_2 and X_3;

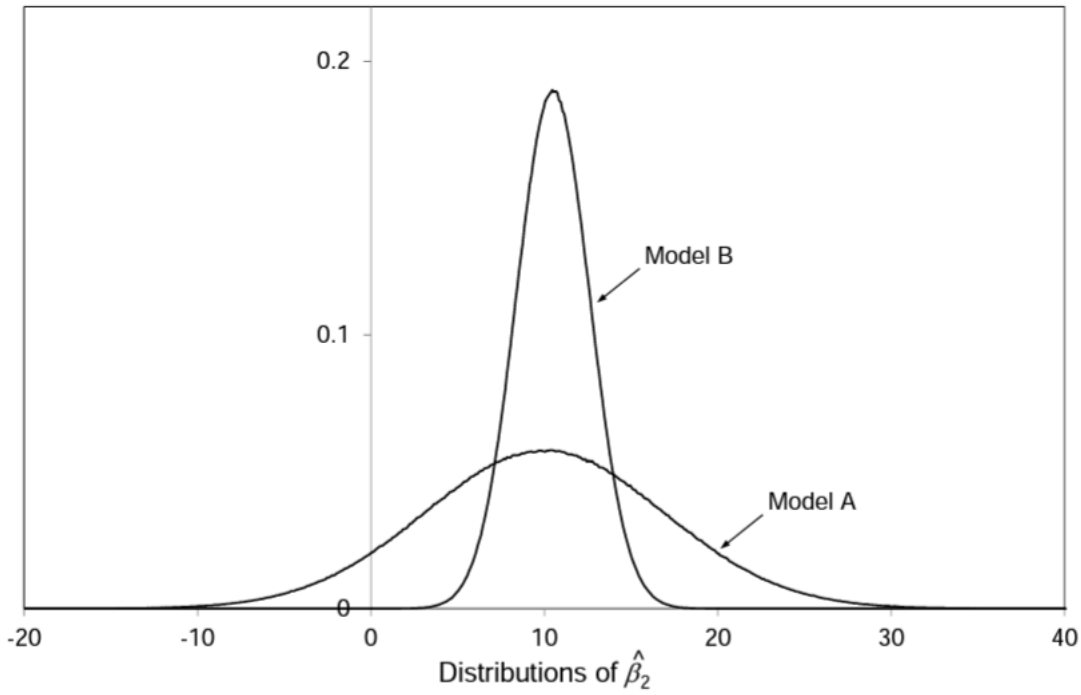
$$Y = 10.0 + 10.0X_2 + 0.5X_3 + u$$

where X_2 is the sequence of integers $1, 2, \cdots, 30$, X_3 is generated from X_2 by adding random numbers, and u is a disturbance term with a normal distribution with mean zero and variance $10,000$. The correlation between X_2 and X_3 is 0.95. The table shows the result of fitting the following regressions for 10 samples:

$$\text{Model A} : \hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3$$
$$\text{Model B} : \hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_2$$

The figure shows the distributions of $\hat{\beta}_2$ for the two models for 10 million samples. In the case of Model A, the distribution of $\hat{\beta}_2$ has mean 10.001 and standard deviation 6.910. For Model B, the mean is 10.500 and the standard deviation is 2.109. Comment on all aspects of the regression results, giving full explanations of what you observe.

	Model A					Model B		
Sample	$\hat{\beta}_2$	s.e.$\left(\hat{\beta}_2\right)$	$\hat{\beta}_3$	s.e.$\left(\hat{\beta}_3\right)$	R^2	$\hat{\beta}_2$	s.e.$\left(\hat{\beta}_2\right)$	R^2
1	10.68	6.05	0.60	5.76	0.5800	11.28	1.82	0.5799
2	7.52	7.11	3.74	6.77	0.5018	11.26	2.14	0.4961
3	7.26	6.58	2.93	6.26	0.4907	10.20	1.98	0.4865
4	11.47	8.60	0.23	8.18	0.4239	11.70	2.58	0.4239
5	13.07	6.07	–3.04	5.78	0.5232	10.03	1.83	0.5183
6	16.74	6.63	–4.01	6.32	0.5966	12.73	2.00	0.5906
7	15.70	7.50	–4.80	7.14	0.4614	10.90	2.27	0.4523
8	8.01	8.10	1.50	7.71	0.3542	9.51	2.43	0.3533
9	1.08	6.78	9.52	6.45	0.5133	10.61	2.11	0.4740
10	13.09	7.58	–0.87	7.21	0.5084	12.22	2.27	0.5081


0.2
0.1
0
Model B
Model A
-20
-10
0
10
20
30
40
Distributions of $\hat{\beta}_2$

2. (Textbook Question 6.8) 10 points

Following is the results of regressing $LGEARN$ on S, EXP, $ASVABC$, $MALE$, $ETHBLACK$, $ETHHISP$. Now we repeat the regression adding AGE. ($LGEARN$ denotes the logged hourly earnings, S represents years of schoolings, EXP represents the total out-of-school work experience (years), $ASVABC$ represents scaled score on a component of the $ASVAB$ test, $MALE$ is a binary variable denoting male, $ETHBLACK$,$ETHHISP$ are binary variables denoting certain ethnicity.)

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   0.977    0.194       5.04  6.62e- 7
## 2 S             0.0954   0.0106      8.99  5.35e-18
## 3 EXP           0.0431   0.00893     4.83  1.81e- 6
## 4 ASVABC        0.0478   0.0283      1.69  9.18e- 2
## 5 MALE          0.195    0.0443      4.41  1.28e- 5
## 6 ETHBLACK     -0.0448   0.0747     -0.600 5.49e- 1
## 7 ETHHISP       0.123    0.0693      1.77  7.72e- 2
```

```
## # A tibble: 8 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   1.29      0.475      2.71  6.94e- 3
## 2 S             0.0985    0.0115     8.57  1.30e-16
## 3 EXP           0.0473    0.0107     4.44  1.12e- 5
## 4 ASVABC        0.0450    0.0286     1.57  1.16e- 1
## 5 MALE          0.194     0.0444     4.36  1.57e- 5
## 6 ETHBLACK     -0.0398    0.0751    -0.530 5.96e- 1
## 7 ETHHISP       0.122     0.0693     1.76  7.91e- 2
## 8 AGE          -0.0132    0.0185    -0.715 4.75e- 1
```

Correlations between AGE and other explanatory variables are given as follows:

```
##                S     EXP  ASVABC    MALE ETHBLACK ETHHISP     AGE
## S         1.0000 -0.5003  0.5338 -0.1852  -0.0891 -0.1215  0.0748
## EXP      -0.5003  1.0000 -0.2119  0.0990  -0.0804  0.0607  0.4165
## ASVABC    0.5338 -0.2119  1.0000 -0.0902  -0.3162 -0.1328 -0.0511
## MALE     -0.1852  0.0990 -0.0902  1.0000  -0.0381 -0.0558 -0.0581
## ETHBLACK -0.0891 -0.0804 -0.3162 -0.0381   1.0000 -0.1299  0.0417
## ETHHISP  -0.1215  0.0607 -0.1328 -0.0558  -0.1299  1.0000 -0.0196
## AGE       0.0748  0.4165 -0.0511 -0.0581   0.0417 -0.0196  1.0000
```

Compare the results of the two regressions.

3. (Textbook Question 5.10) 10 points

The regression model looks as follows:

$$\log(\text{EARNINGS})_i = \beta_0 + \beta_1 S_i + \beta_2 EXP_i + \beta_3 MALE + \beta_4 COLLBARG + u_i.$$

Following is the results of regressing logged hourly earnings, denoted by log(EARNINGS), on S, EXP, $MALE$, and $COLLBARG$. (log(EARNINGS) denotes the logged hourly earnings, S represents years of schoolings, EXP represents the total out-of-school work experience (years), $MALE$ is a binary variable denoting male.)

Does belonging to a union have an impact on earnings? In the output below, $COLLBARG$ is a dummy variable defined to be 1 for workers whose wages are determined by collective bargaining and 0 for the others. Provide an interpretation of the regression coefficients and perform appropriate statistical tests.

```
## # A tibble: 5 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   1.04     0.197        5.29 1.87e- 7
## 2 S             0.0932   0.0101       9.19 1.11e-18
## 3 EXP           0.0423   0.00940      4.50 8.61e- 6
## 4 MALE          0.172    0.0452       3.79 1.67e- 4
## 5 COLLBARG      0.258    0.0631       4.08 5.19e- 5
```

4. (Textbook Question 5.19) 10 points

Is the effect of education on earnings different for members of a union? In the output below, $COLLBARG$ is a dummy variable defined to be 1 for workers whose wages are determined by collective bargaining and 0 for the others. $SBARG$ is a slope dummy variable defined as the product of S and $COLLBARG$. Provide an interpretation of the regression coefficients, comparing them with those in question 3, and perform appropriate statistical tests.

```
## # A tibble: 6 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  1.03      0.205       5.05  6.24e- 7
## 2 S            0.0937    0.0108      8.66  6.65e-17
## 3 EXP          0.0423    0.00941     4.49  8.75e- 6
## 4 MALE         0.171     0.0454      3.78  1.78e- 4
## 5 COLLBARG     0.298     0.357       0.835 4.04e- 1
## 6 SBARG       -0.00261   0.0227     -0.115 9.08e- 1
```