AEM 7130: Dynamic Optimization

Spring 2019: Monday 4:30-7:00
Room: Warren 113
`http://github.com/AEM7130`
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Course Overview

Course Objective: The objective of this course is to familiarize you with the fundamentals of solving dynamic models through computation. In this course you should learn why we need computational methods for certain types of problems, the theory behind the methods, and most importantly, how to use them in practice. The course begins with general basics of computing and optimization. Next, the course covers some basic dynamic theory and then goes over a selection of methods for computing solutions to dynamic models either numerically or empirically. We will conclude with how to use high performance computing resources like computing clusters or graphical processing units. The beginning of the course is heavy on theory to understand what is happening inside your machine when solving numerical models. As we begin studying techniques to solve economic problems you will also apply them in practice.

I will be teaching the class in Julia. Julia is becoming widely used in computational economics because it's open source, it has many packages to employ the methods we will learn and practice, and it's fast and intuitive. **Please set up GitHub (`https://github.com`) and JuliaBox (`https://juliabox.com`) accounts before class starts.** JuliaBox allows you to compute in Julia using cloud (and multi-core) resources for free. Much of what we do can be easily ported to R, Python, MATLAB, and C.

Prerequisites: ECON 6090 and ECON 6170.

Programs: If you want to do the computing directly on your machine instead of on JuliaBox you will want to install the following programs

- Jupyter/Anaconda: `https://www.anaconda.com/download/`[*]
- Julia (command line) or JuliaPro (command line and IDE): `https://julialang.org/downloads/`

Readings: Some theory on dynamics will draw from Karp and Traeger (2013).[†] Nocedal and Wright (2006) is highly useful as a detailed reference for optimization. Judd (1998) and Miranda and Fackler (2002) take a more detailed look at the fundamental numerical methods in economics. Judd (1998), Miranda and Fackler (2002) and Nocedal and Wright (2006) are available as eBooks in the library and Karp and Traeger (2013) will be available on blackboard or from the authors' websites. Please read *Learn Julia the Hard Way* or go over the first few QuantEcon Julia lectures (`https://quantecon.org`) for a brief introduction to coding in Julia. The remainder of the required readings will be from journal articles or excerpts from texts which will be accessible online and posted on GitHub a week before class.

[*]This will also install Python if you want to use that language.
[†]Stokey (1989) and Caputo (2005) provide a deeper treatment of dynamic programming and optimal control.

Grading: 10% of your grade is class participation, 10% is your presentation of a numerical paper, 10% is your final project proposal, 20% is the final project, and 50% is the problem sets.

Important Dates:
Final project proposals due: March 18
Final project presentations: May 6
Final project paper due: May 7

Office Hours: Tuesday 1:30-3:00 in Warren 462.

Contacting Me: Please put AEM 7130 in the subject line of all e-mails and I will respond within 24 hours.

Assignments

1. There will be five problem sets. You must submit your code on GitHub (url above) to your group's repository. We will learn how to use Git during class. You may work in a group of **three or fewer people**. **You must submit your problem set solutions as a well-commented (and already executed) Jupyter notebook in either Julia, Python, or R.** Each group should turn in one assignment with all members' names in the first cell of the notebook.

2. There is a final project for the course, due at the end of the semester, where each student will submit the beginning of a numerically-driven research paper. A proposal of the final project is due at about the halfway point of the course. During the final week of class, each student will present their completed work which should have a first-take at a numerical/empirical model and preliminary results. The paper is due the day after the final class. It should be at least 10 pages including tables and graphs and should:

    - Have an introduction that clearly states the economic question you are answering, frames your research in the context of the existing literature, and tells the reader what you are doing to advance economic knowledge.
    - Analytically develop the model, provide proofs for theoretical results if there are any.
    - Describe how you solve the model.
    - Have preliminary results.

3. Starting near the middle of the course, one student a week will present either a paper that either applies methods we have learned in a previous week, or extends methods we have previously learned. More information will come later in the course.

Tentative Course Schedule

Day 1: Introduction to Computation

Theory: Matrix Inversion, Differentiation, Integration, Storage of Numbers, Truncation, Rounding, Error Propagation

Miranda and Fackler (2002, Chapters 1, 2, and 5); Judd (1998, Chapters 2, 3 and 7)

Day 2: Coding and Version Control

Application: Git, Types, Control Flow, Style, Making Things Fast

Learn Julia the Hard Way, QuantEcon lectures

Day 3: Rootfinding and Optimization

Theory: (Quasi-)Newton, Nelder-Mead, Line Search, Trust Region, Global Solvers

Miranda and Fackler (2002, Chapters 3 and 4); Judd (1998, Chapter 4 and 5); Nocedal and Wright (2006, Chapters 2-6)

Day 4: Discrete Time Dynamic Programming

Theory: Markov Chains, Bellman's Principle of Optimality, Linear-Quadratic Models

Ljungqvist and Sargent (2004); Adda and Cooper (2003)

Days 4,5: Numerical Methods for Discrete Time Models

Theory: Discretization, Projection, Perturbation

Aruoba et al. (2006); Cai and Judd (2014); Fernández-Villaverde et al. (2016); Lemoine and Rudik (2017a)

Application: Monetary Policy, Climate Change, Bioeconomic Modeling, Agriculture

Cogley et al. (2007); Springborn and Sanchirico (2013); Lemoine and Traeger (2014); Traeger (2014); Cai et al. (2015); Livingston et al. (2015)

Day 6: Continuous Time Optimal Control

Theory: Maximum Principle, Hamiltonians

Judd (1998, Chapter 10); Caputo (2005)

Application: Climate Change, Resource Depletion, Shallow Lakes, Antibiotics

Goulder and Mathai (2000); Laxminarayan and Brown (2001); Maler et al. (2003); Venables (2014); Lemoine and Rudik (2017b); Anderson et al. (2018)

Days 6,7: Numerical Methods for Continuous Time Deterministic Models

Theory: Backwards Shooting/Integration, Relaxation Algorithm

Judd (1998, Chapter 10); Brunner and Strulik (2002); Trimborn et al. (2008)

Application: Climate Change, Resource Depletion, Shallow Lakes, Antibiotics

Goulder and Mathai (2000); Laxminarayan and Brown (2001); Maler et al. (2003); Venables (2014); Lemoine and Rudik (2017b); Anderson et al. (2018)

Days 8-10: Some Advanced Methods for Numerical Dynamic Models

Theory: Modified Policy Iteration, Sparse Grids, Endogenous Grids, Adaptive Grids, Stochastic Simulation, Precomputation, Envelope Condition Methods, Non-linear Certainty Equivalent Approximation, Multicollinearity, Lower Bounds on Errors, Euler Equation Errors, χ^2 Errors

Smolyak (1963); Puterman and Shin (1978); Den Haan and Marcet (1994); Santos and Vigo-Aguiar (1998); Santos (2000); Carroll (2006); Winschel and Kratzig (2010); Malin et al. (2011); Peralta-alva and Santos (2014); Maliar and Maliar (2014); Judd et al. (2014, 2015); Maliar and Maliar (2015); Judd et al. (2016); Brumm and Scheidegger (2017); Cai et al. (2017); Judd et al. (2017); Cai (2018); Coleman et al. (2018)

Day 10-12: Empirical Dynamic Models and Some Advanced Methods

Theory: Nested Fixed Point, Mathematical Programming with Equilibrium Constraints, Bounding Counterfactuals, Bray's Method

Rust (1987); Dubé et al. (2012); Su and Judd (2012); Reguant (2016); Bray (2018a,b)

Day 12: Efficient Methods for Expectations (in High Dimensions)

Theory: Markov Chain Monte Carlo, Hamiltonian Monte Carlo, Monomial Rules

Chib and Greenberg (1995, 1996); Skrainka and Judd (2011); Betancourt (2017)

Application: Non-Conjugate Learning, Uncertainty Shocks, BLP without Monte Carlo

Skrainka and Judd (2011); Lieli and Springborn (2012); Orlik and Veldkamp (2014)

Day 13: High Performance Computing: Computing Clusters and GPUs

Theory: Aldrich et al. (2011); Aldrich (2014); Cai et al. (2014)

Video: https://www.youtube.com/watch?v=ehUb-S6Bxnk

Day 13: Final Project Presentations

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